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On the local multiset dimension of graph with homogenous pendant edges

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Abstract. Let G be a connected graph with E as edge set and V as vertex set. $r_m(v|W) = \{d(v, s_1), d(v, s_2), \dots, d(v, s_k)\}$ is the multiset representation of a vertex v of G with respect to W where $d(v, s_i)$ is a distance between of the vertex v and the vertices in W for k -ordered set $W = \{s_1, s_2, \dots, s_k\}$ of vertex set G . If $r_m(v|W) \neq r_m(u|W)$ for every pair u, v of adjacent vertices of G , we called it as local resolving set of G . The minimum cardinality of local resolving set W is called local multiset dimension. It is denoted by $\mu_l(G)$. $H_i \cong H$, for all $i \in V(G)$. If $H \cong K_1$, $G \odot H$ is equal to the graph produced by adding one pendant edge to every vertex of G . If $H \cong mK_1$ where mK_1 is union of trivial graph K_1 , $G \odot H$ is equal to the graph produced by adding one m pendant edge to every vertex of G . In this paper, we analyze the exact value of local multiset dimension on some graphs with homogeneous pendant edges.

1. Introduction

We concern on simple and undirected graphs. Metric dimension is one of the topic in graph theory which has many applications in real life. Navigation system is one of application that used metric dimension topic. By using metric dimension, we can make an effective coordinate for navigation system. Suppose, $W = \{s_1, s_2, \dots, s_k\}$ be the ordered set of vertices of a graph G . We can called W as the resolving set of G if the distinct vertices of G of the set W have distinct representations with respect to W , where the representation of v with respect to W is k -vector $r(v|W) = (d(v, s_1), d(v, s_2), \dots, d(v, s_k))$. The metric dimension of G is the minimum cardinality of resolving set W of G is called, denoted by $dim(G)$ [9, 8].

The development of this theory has been carry out extensively. There are many types of metric dimension development, such as: partition dimension, edge metric dimension, star partition dimension, multiset dimension, etc. We can see the detail of these topics in [5, 9, 1]. In this paper, we develop a new study about multiset dimension. Before we go on to the development of multiset dimension, let firstly we discuss the definion of multiset dimension. Suppose $W = \{s_1, s_2, \dots, s_k\}$ is a subset of vertex set $V(G)$. Let we define $d(v, s_i)$ as the distance between of v and the vertices in W . $r_m(v|W) = \{d(v, s_1), d(v, s_2), \dots, d(v, s_k)\}$ is the representation multiset of a vertex v of G with respect to W . We say that the resolving set W as a resolving set of G if $r_m(v|W) \neq r_m(u|W)$ for every pair u and v . The minimum resolving set W is a multiset basis of G . The multiset dimension is the cardinality of multiset basis of



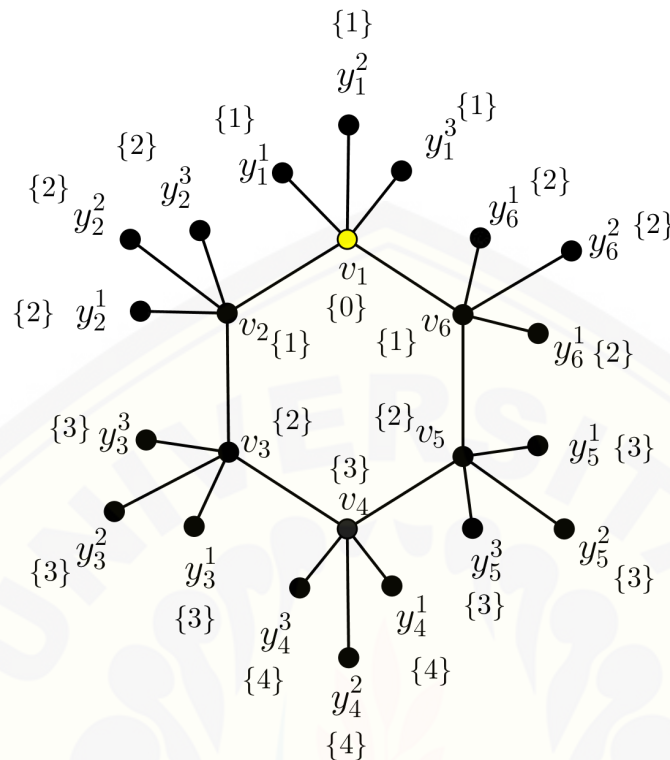


Figure 1. A cycle graph order 6 with 3 homogenous pendant edges $C_6 \odot 3K_1$ with local multiset dimension 1

graph G . It is denoted by $md(G)$. We can see the explanation of this study in [12]. Now, we develop a new study in metric dimension namely local multiset dimension. This study was the combination of multiset dimension and local metric dimension.

For more detail about definition and graph terminology, we can see [3, 4, 7, 10]. If we consider the ordered sets W of any vertices in graph G for which any two vertices of G having the same representation with respect to W are not adjacent in G . W is called a local resolving set of G if $r(u|W) \neq r(v|W)$ for every pair u, v of adjacent vertices of G . The local metric dimension of G is the minimum cardinality of local resolving set is. We denoted it by $ldim(G)$ [6]. There are many studies about the local metric dimension. This topic has been studied by [6] and some related topic in the developoment of metcur dimension has been studied by [5, 15, 2]. In this paper, we are focused on determining the local multiset dimension of graph with homogenous pendant edges. If $H \cong mK_1$ where mK_1 is union of trivial graph K_1 , $G \odot H$ is equal to the graph produced by adding one m pendant edge to every vertex of G . In this paper, we search some exact value of local multiset dimension of some graph with homogeneous pendant edges.

2. Local Multiset Dimension

In this section, we will discuss about the definiton and example of local multiset dimension. The local multiset dimension is the development of metric dimension study. This topic was introduced by Ridho in [14]. The definition of local multiset dimension is as follows:

Definition 1.1 : Suppose $W = \{s_1, s_2, \dots, s_k\}$ is a subset of vertex set $V(G)$. $r_m(v|W) = \{d(v, s_1), d(v, s_2), \dots, d(v, s_k)\}$ is the representation multiset of a vertex v of G with respect to W , denoted by $d(v, s_i)$. The resolving set W is a local resolving set of G if $r_m(v|W) \neq r_m(u|W)$.

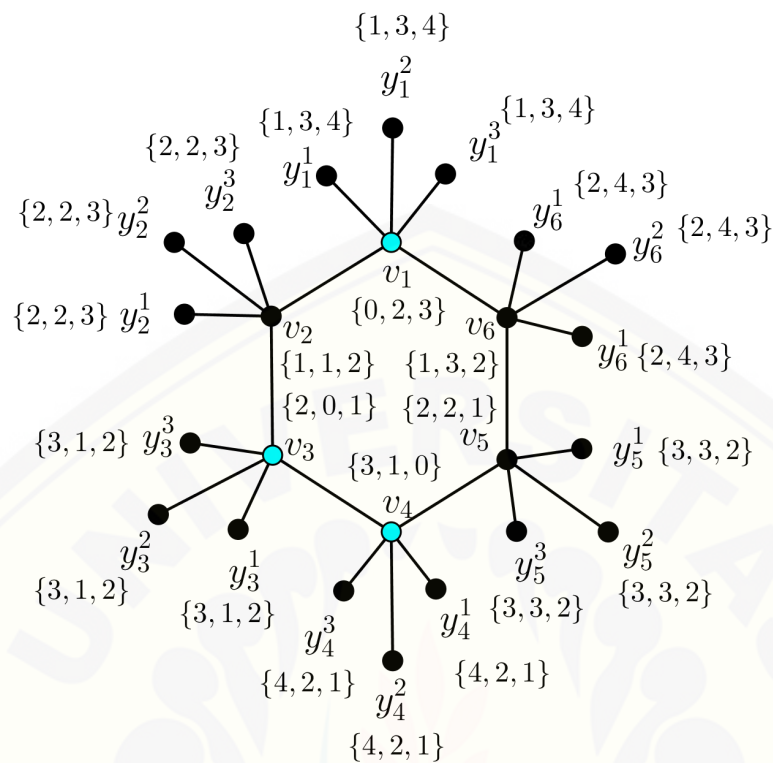


Figure 2. A cycle graph order 6 with 3 homogenous pendant edges $C_6 \odot 3K_1$ with multiset dimension 3

The minimum cardinality of local resolving set W is called as local multiset dimension. It is denoted by $\mu_l(G)$.

We can see the illustration of local multiset dimension concept in 1 and the illustration of multiset dimension concept Figure 2. Based of Figure 2 and 1, we can see the differences between multiset dimension and local multiset dimension. Based on figure 2, the resolving set of $C_6 \odot 3K_1$ is $W = \{v_1, v_3, v_4\}$. Since we have $W = \{v_1, v_3, v_4\}$, thus the cardinality of multiset dimension of $C_6 \odot 3K_1$ is 3 or we can write $md(G) = 3$. Consider to the representations of $v \in V(C_6 \odot 3K_1)$ with respect to W in figure 2, we can see that the representation of each vertex is distinct. While in local multiset dimension of $C_6 \odot 3K_1$, we have local resolving set v_1 or $W = \{v_1\}$. Therefore, we get the representation of the vertices of $C_6 \odot 3K_1$ with respect to W are as follows:

$$\begin{array}{lll}
 r_m(v_1|\Pi) = \{0\}, & r_m(v_2|\Pi) = \{1\}, & r_m(v_3|\Pi) = \{2\} \\
 r_m(v_4|\Pi) = \{4\}, & r_m(v_5|\Pi) = \{2\}, & r_m(v_6|\Pi) = \{1\} \\
 r_m(v_1^j|\Pi) = \{1\}, & r_m(v_2^j|\Pi) = \{2\}, & r_m(v_3^j|\Pi) = \{3\} \\
 r_m(v_4^j|\Pi) = \{4\}, & r_m(v_5^j|\Pi) = \{3\}, & r_m(v_6^j|\Pi) = \{2\}
 \end{array}$$

with the value of j is $1 \leq j \leq 3$. We can see that the representations of adjacent vertices in cycle graph order 6 with homogenous pendant edges $v \in V(C_6 \odot 3K_1)$ with respect to W are distinct. Thus, we can conclude that $\mu_l(C_6 \odot 3K_1) = 1$.

3. Main results

Before we analyze the exact value of local multiset dimension of a graph with homogeneous pendant edges, let firstly we discuss about the properties of local multiset dimension of complete

graph and tree graph. The following theorems are useful for determining the local multiset dimension of a graph with homogeneous pendant edges in the next theorem. These theorem are already discussed and proved by Ridho Et. Al in. [14]. Here is the theorem of local multiset dimension of complete graph:

Theorem 3.1 *Let K_n be a complete graph with $n \geq 3$, the local multiset dimension of K_n is ∞*

The next theorem is local multiset dimension of complete k -ary tree of height h :

Lemma 1 *Let T_n be a tree graph of order n , we have $\mu_l(T) \geq 1$*

Now, we go on to the new results about local metric dimension of graph with homogeneous pendant edge which have been discovered. Here are the results:

Theorem 3.2 *Let $C_n \odot mK_1$ be a cycle graph with homogeneous pendant edges. For $n \geq 4$, the local multiset dimension of $C_n \odot mK_1$ is*

$$\mu_l(C_n \odot mK_1) = \begin{cases} 1, & \text{if } n \text{ is even} \\ 2, & \text{if } n \text{ is odd} \end{cases}$$

Proof. The cycle $C_n \odot mK_1$ is a cyclic graph with $n > 3$. The vertex set $V(C_n \odot mK_1) = \{v_i\} \cup \{y_i^j\}; 1 \leq i \leq n; 1 \leq j \leq m$ and edge set $E(C_n \odot mK_1) = \{v_1v_n, v_iv_{i+1}; 1 \leq i \leq n-1\} \cup \{v_iy_i^j; 1 \leq i \leq n-1; 1 \leq j \leq m\}$. The cardinality of vertex set is $n + nm$ and the cardinality of edge set is $n + nm$. The proof of this theorem is divided into two cases as follows.

Case 1: For n is even, Based on theorem 1, we have if K_n be a complete graph with $n \geq 3$, the local multiset dimension of K_n is $\mu_l(K_n) = \infty$. We know that $C_n \odot mK_1$ is not a complete graph. We have known that the lower bound of local multiset dimension of cycle graph with homogeneous pendant edges for n even is 1 or we can write $\mu_l(C_n \odot mK_1) \leq 1$. Next, we shold show that the upper bound of local multiset dimension of cycle is 1 or $\mu_l(C_n \odot mK_1) \leq 1$. Let $W = \{v_1\}$, the representation of vertices $v \in V(C_n \odot mK_1)$ respect to W as follows.

$$\begin{aligned} r(v_1|W) &= \{0\} \\ r(v_i|W) &= \{i-1\}; 2 \leq i \leq \frac{n}{2} + 1 \\ r(v_i|W) &= \{n-i+1\}; \frac{n}{2} + 2 \leq i \leq n \\ r(y_1^j|W) &= \{1\} \\ r(y_i^j|W) &= \{i\}; 2 \leq i \leq \frac{n}{2} + 1 \\ r(y_i^j|W) &= \{n-i\}; \frac{n}{2} + 2 \leq i \leq n \end{aligned}$$

It can be seen that $r_m(v_i|W) \neq r_m(v_{i+1}|W)$ with v_i and v_{i+1} are adjacent for $1 \leq i \leq n-1$. It also can be seen that $r_m(y_i^j|W) \neq r_m(y_i^{j+1}|W)$ with y_i^{j+1} and y_i^j are adjacent for $1 \leq j \leq n$. Thus, we get the upper bound of local multiset dimension of cycle $C_n \odot mK_1$ is 1 or $\mu_l(C_n \odot mK_1) \leq 1$. It concluded that $\mu_l(C_n \odot mK_1) = 1$ for n is even.

Case 2: If n is element of odd number, It will be analyzed that lower bound of the local multiset dimension of $C_n \odot mK_1$ is 2 or $\mu_l(C_n \odot mK_1) \geq 2$. By using contradiction, let $\mu_l(C_n \odot mK_1) < 2$, suppose the local resolving set is 1 or $W = \{u\}$. Thus, we will have some conditions as follow:

- a) If we have the local resolving set of $C_n \odot mK_1$ $u \in W$ is in pendant vertex y_i^j , then $r_m(x_{\frac{n+1}{2}}|W) = r_m(x_{\frac{n+1}{2}+1}|W) = \{\frac{n+1}{2}\}$. There are some adjacent vertices which have same representation. it is a contradiction.
- b) The next condition, let $u \in W$ are in cycle graph. We know that there exist path P_m , where m is even and connected with vertex u . Suppose that the vertices in P_m be $v_1, v_2, \dots, v_{n-1} \in V(P_m)$ for $n \in \mathbb{Z}^+$.
- c) Based on point b, we can see that $r_m(v_{\frac{m}{2}}|W) = r_m(v_{\frac{m}{2}+1}|W) = \{\frac{m}{2}\}$. We know that $v_{\frac{m}{2}}$ adjacent with $v_{\frac{m}{2}+1}$. Therefore, it is a contradict with our definition of local multiset dimension.

Based on the analysis above, it can be seen that the lower bound of local multiset dimension of $C_n \odot mK_1$ is 2 or $\mu_l(C_n \odot mK_1) \geq 2$. The next proof is the proof of $C_n \odot mK_1$ local multiset dimension upper bound. We will show the upper bound of the local multiset dimension of $C_n \odot mK_1$ is 2 or $\mu_l(C_n \odot mK_1) \leq 2$. let $W = \{y_1, x_2\}$ be the local resolving set of $C_n \odot mK_1$. The representation v respect to W in $C_n \odot mK_1$ are as follows:

$$\begin{aligned}
 r_m(x_1|W) &= \{1, 1\} \\
 r_m(x_2|W) &= \{1, 0\} \\
 r_m(x_i|W) &= \{i, i - 2\}; 3 \leq i \leq \frac{n-1}{2} + 2 \\
 r_m(x_i|W) &= \{n - i + 2, n - i + 2\}; \frac{n-1}{2} + 3 \leq i \leq n \\
 r_m(y_1^j|W) &= \{2, 2\} \\
 r_m(y_2^j|W) &= \{2, 1\} \\
 r_m(y_i^j|W) &= \{i+, i - 1\}; 3 \leq i \leq \frac{n-1}{2} + 2 \\
 r_m(y_i^j|W) &= \{n - i + 3, n - i + 3\}; \frac{n-1}{2} + 3 \leq i \leq n
 \end{aligned}$$

Theorem 3.3 Let $P_n \odot mK_1$ be a path graph with homogenous pendant edges. For $n \geq 3$, the local multiset dimension of $P_n \odot mK_1$ is 1.

Proof. The path $P_n \odot mK_1$ is a tree graph. The vertex set $V(P_n \odot mK_1) = \{v_1, v_2, \dots, v_n\} \cup \{y_i^j\}$ and edge set $E(P_n \odot mK_1) = \{v_i v_{i+1}; v_i y_i^j; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$. The cardinality of vertex set is $n + m$ or $|V(P_n \odot mK_1)| = n + nm$ and the cardinality of edge set respectively is $n - 1 + nm$ or $|E(P_n \odot mK_1)| = n - 1 + nm$.

According to Lemma 1, the lower bound of local multiset dimension of tree graph T is $\mu_l(T) \geq 1$. We know that $P_n \odot mK_1$ is tree graph such that $\mu_l(P_n \odot mK_1) \geq 1$. Thus, the local multiset dimension of $P_n \odot mK_1$ attain the lower bound. The next step, we will prove the upper bound of local multiset dimension of path $P_n \odot mK_1$ is 1 or $\mu_l(P_n \odot mK_1) \leq 1$. Let we assume that $W = \{v_1\}$. We got the representation of vertices $v \in V(P_n \odot mK_1)$ respect to W are as follow:

$$\begin{aligned}
 r_m(v_1|W) &= \{0\} \\
 r_m(v_i|W) &= \{i - 1\}; 2 \leq i \leq n \\
 r_m(y_1^j|W) &= \{1\} \\
 r_m(y_i^j|W) &= \{i\}; 2 \leq i \leq n
 \end{aligned}$$

We can see that $r_m(v_i|W) \neq r_m(v_{i+1}|W)$. Therefore, we attain the upper bound of local multiset dimension of $P_n \odot mK_1$ is $\mu_l(P_n \odot mK_1) \leq 1$. Since we have proved the lower bound and upper bound of local multiset dimension of $P_n \odot mK_1$, it can be concluded that $\mu_l(P_n) = 1$

Theorem 3.4 *Let $S_n \odot mK_1$ be a star graph with homogenous pendant edges. For $n \geq 3$, the local multiset dimension of $S_n \odot mK_1$ is 1.*

Proof. The star with homogenous pendant edges $S_n \odot mK_1$ is a tree graph with $2n$ vertices. The vertex set $V(S_n \odot mK_1) = \{v_1, v_2, \dots, v_n\} \cup \{v_i^j\} \cup \{c\}$ and edge set $E(S_n \odot mK_1) = \{Cv_i; v_i y_i^j; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$. The cardinality of vertex set is $2n + 1$ or $|V(S_n \odot mK_1)| = 2n + 1$ and the cardinality of edge set respectively is $|E(S_n \odot mK_1)| = 2n + 1$.

Based on Lemma 1, we have the lower bound of local multiset dimension of tree graph T is $\mu_l(T) \geq 1$. We know that $S_n \odot mK_1$ is tree graph such that $\mu_l(S_n \odot mK_1) \geq 1$. In other word, the local multiset dimension of $S_n \odot mK_1$ attain the lower bound. Furthermore, we will analyze the local multiset dimension of $S_n \odot mK_1$ upper bound. We wil show that $\mu_l(S_n \odot mK_1) \leq 1$. By assuming $W = \{c\}$, we get the representation of vertices $v \in V(S_n \odot mK_1)$ respect to W are as follow:

$$r_m(v_i|W) = \{1\}; 1 \leq i \leq n$$

$$r_m(v_i^j|W) = \{2\}; 1 \leq i \leq n; 1 \leq j \leq m$$

We can see that $r_m(v_i|W) \neq r_m(v_{i+1}|W)$. Hence, it is proved that the upper bound of local multiset dimension of $S_n \odot mK_1$ is 1 or $\mu_l(S_n \odot mK_1) \leq 1$. Since we have $\mu_l(S_n \odot mK_1) \leq 1$ and $\mu_l(S_n \odot mK_1) \geq 1$, it can be concluded that $\mu_l(S_n \odot mK_1) = 1$.

Theorem 3.5 *Let $K_n \odot mK_1$ be a complete graph with homogenous pendant edges. For $n \geq 3$, the local multiset dimension of $K_n \odot mK_1$ is ∞*

Proof: Let $K_n \odot mK_1$ be a complete graph with homogenous pendant edges with $n + m$ vertices. The vertex set $V(K_n \odot mK_1) = \{v_1, v_2, \dots, v_n\} \cup \{v_i^j; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$ and edge set $E(K_n \odot mK_1) = \{v_i v_{i+k}; v_i y_i^j; 1 \leq i \leq n, 1 \leq k \leq n - i\}$. The vertex set cardinality of $K_n \odot mK_1$ is $n + m$ or $|K_n \odot mK_1| = n + m$ and edge set cardinality respectively is $|E(K_n)| = \frac{n(n-1)}{2} + nm$.

We use contradiction in proving this theorem. $K_n \odot mK_1$ consist of complete graph by adding one pendant egdges in every vertex in complete graph. In order to prove this theorem, let we consider the complete graph. Suppose that all vertices in W have distance 1 and the local resolving set of complete graph K_n is W . Thus, we will have the following conditions:

- If we take $W = \{v_1\}$, then $r(v_1|W) = \{0\}$ and $r_m(v_2|W) = r_m(v_3|W) = \dots = r_m(v_{n-1}|W) = r_m(v_n|W) = \{1\}$, we know that v_2, v_3, \dots, v_n are adjacent such that it is a contradiction.
- If we take $W = \{v_1, v_2\}$, then $r(v_1|W) = r(v_2|W) = \{0, 1\}$ and $r_m(v_3|W) = \dots = r_m(v_{n-1}|W) = r_m(v_n|W) = \{1^2\}$, we know that v_3, \dots, v_n are adjacent such that it is a contradiction.
- If we take $W = \{v_1, v_2, \dots, v_k\}$ for $2 \leq k \leq n - 1$, then $r(v_1|W) = \dots = r(v_k|W) = \{0, 1^{k-1}\}$ and $r_m(v_{k+1}|W) = \dots = r_m(v_n|W) = \{1^k\}$, we know that v_{k+1}, \dots, v_n are adjacent such that it is a contradiction.
- If W are in pendant v_i^j , then there will be exist some adjacent vertices v_i and v_{i+1} in complete graph who have the same resolving set, it is a contradiction

Based on the analysis above, we can see that W is not a local resolving set of complete graph $K_n \odot mK_1$. It is conclude that $\mu_l(K_n \odot mK_1) = \infty$.

Theorem 3.6 Let $C_{n,m} \odot mK_1$ be a caterpillar graph with homogenous pendant edges with $n \geq 3$ and $m \geq 2$, the local multiset dimension of $C_{n,m} \odot mK_1$ is $\mu_l(C_{n,m} \odot mK_1) = 1$

Proof: The caterpillar graph with homogenous pendant edges $C_{n,m} \odot mK_1$ is a tree graph with $nm + n$ vertices. The vertex set $V(C_{n,m} \odot mK_1) = \{v_1, v_2, \dots, v_n\} \cup \{v_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_{n,m} \odot mK_1) = \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{v_i v_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$. The cardinality of vertex set and edge set, respectively are $|V(C_{n,m} \odot mK_1)| = n + nm + mk + nk$ and $|E(C_{n,m})| = nm + n + nk + mk - 1$.

Based on Lemma 1 that the lower bound of local multiset dimension of tree graph T is $\mu_l(T) \geq 1$. We know that $C_{n,m} \odot mK_1$ is tree graph such that $\mu_l(C_{n,m} \odot mK_1) \geq 1$. However, we attain the sharpest lower bound. Furthermore, The upper bound of local multiset dimension of $C_{n,m} \odot mK_1$ is $\mu_l(C_{n,m} \odot mK_1) \leq 1$. Suppose $W = \{v_1\}$, the representation of vertices $v \in V(C_{n,m} \odot mK_1)$ respect to W is

$$r_m(v_1|W) = \{0\}$$

$$r_m(v_i|W) = \{i-1\}; 2 \leq i \leq n$$

$$r_m(v_{i,j}|W) = \{i\}; 1 \leq i \leq n, 1 \leq j \leq m$$

$$r_m(v_{i,j,k}|W) = \{i+1\}; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq l$$

Thus, we obtain the upper bound of local multiset dimension of $C_{n,m} \odot mK_1$ is $\mu_l(C_{n,m} \odot mK_1) \leq 1$. We conclude that $\mu_l(C_{n,m} \odot mK_1) = 1$.

4. Concluding Remarks

In this paper we have already proved the local multiset dimension of graph with homogeneous pendant edges namely cycle with homogeneous pendant edges, path with homogeneous pendant edges, star with homogeneous pendant edge, complete graph with homogeneous pendant edges, and caterpillar with homogeneous pendant edges. Since there were many kind of graphs family, we have the following open problems

Open Problem 1 Analyze the local multiset dimension of another graph operations.

Open Problem 2 Investigate the general lower bound of local multiset dimension.

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