MATERIALS AND METHODS

A block model was developed based on the geologist model. The geologists generated multiple layers of coal deposit (**FIGURE 1b**) that consist of 28 lithological codes; 4 coal codes (assigned "1") and 24 non-coal codes (assigned "0"). There were 268 drill holes inside the case study area that contained a calorific value (CV), ash content, sulfur content, and moisture content data (hard conditioning data). The geologist model (training image) consisted of 95 x 70 x 165 blocks for 20 x 20 x 10 m block dimension. **FIGURE 1** represents the coal body model developed by the geologist, and **TABLE 1** represents the technical and economic parameters of this research. The difference between overburden (OB) and coal mining cost was mainly because of the different mining methods that affected the selection of the loading and hauling equipment.

(a)

(b)

FIGURE 1. The cross-section of the coal mine pit with coal seams (a) and the coal body model (b)

TABLE 1. The technical and economic parameters

No	Technical Properties	Value	Unit
1	Block Size	20×20×20	m
2	Number of Blocks	113.05	
3	Mining Capacity	45,000,000	tonne/year
4	Slope Constraint	45	degree
5	Mining Period	10	year
6	Mining Recovery	95	%
7	OB Specific Gravity	2.1	tonne/m³
8	Coal Specific Gravity	1.3	tonne/m³

No	Economical Properties	Value	Unit
1	OB Mining Cost	3	$/tonne
2	Coal Mining Cost	0.75	$/tonne
3	Coal Washing Cost	1.4	$/tonne
4	Selling Cost	2	% of Selling Price

5	Annual Discounted Rate	9.5	%

The mean-reversion (Ornstein-Uhlenbeck (OU)) process was extensively applied to simulate interest rates and commodity price [6, 7, 8, 9]. Charles and Darné [10] mentioned the requisite of using this model. The model was chosen to simulate coal price since its capability to represent parsimony or reduced-form together [10, 11, 12, 13]. Furthermore, Lucia and Schwartz [11]and Schwartz [14] stated that the mean-reversion process was one of the most tested stochastic processes for coal prices. Valdivieso et al. [15] and Smith [16] mentioned that the mean-reversion process fully supported the idea of supply and demand economic law; that is when commodity prices are extortionate, the demand for a particular commodity will be lower, and this condition makes the supply of the particular commodity will be abundant, and vice versa. In the end, the prices will follow their long-term mean. With the stochastic differential equation below, the OU process could be extended to simulate coal price:

$$dC_t = \gamma(\rho - C_t)dt + \sigma dW_t \tag{1}$$

$$C_t - C_{t-1} = \gamma(\rho - C_{t-1})\Delta t + \sigma dW_t \tag{2}$$

For time t, C_t is defined as the market coal price. The Eq. 1 consists of two parts, the first one, $\gamma(\rho - C_t)dt$ is defined as a drift component. The mean-reversion speed of the stochastic parameters is measured by variable γ. It indicates how fast the price revert to their long-term mean ρ. The part of σdW_t is complying with the standard Brownian motion. The volatility of the price is estimated by variable σ. The OU process states that the price fluctuations will be close to their mean, and temporarily, there will be price peaks at some time. Gillespie [15] mentioned that Eq. 2 will be valid only if Δt is small enough. He proposed for any value of Δt; the equation will be modified as shown below:

$$C_t = e^{-\gamma\Delta t}C_{t-1} + (1 - e^{-\gamma\Delta t})\rho + \sigma\sqrt{\frac{(1-e^{-2\gamma\Delta t})}{2\gamma}}dW_t \tag{3}$$

Valdivieso [15] and Smith [16] examined the estimation of OU process parameters. Both least square regression and maximum likelihood were widely used and accurate to estimate σ and ρ (following the unbiased estimator and low standard deviation) but not for γ [17]. He confirmed that maximum likelihood estimated $\hat{\sigma}$ and $\hat{\rho}$ close to the true value of σ and ρ. Phillips and Yu [18] proposed to incorporate the jackknife technique with the maximum likelihood for the better estimation process.

$$\rho = \frac{C_y C_{xx} - C_x C_{xy}}{n(C_{xx-} - C_{xy}) - (C_x^2 - C_x C_y)} \tag{4}$$

$$\gamma = -\frac{1}{\Delta t}\log\frac{C_{xy} - \rho C_x - \rho C_y + n\rho^2}{C_{xx}2\rho C_x + n\rho^2} \tag{5}$$

$$\alpha = e^{-\gamma\Delta t} \tag{6}$$

$$\sigma = \sqrt{\left(\frac{1(C_{yy} - 2\alpha C_{xy} + \alpha^2 C_{xx} - 2\rho(1-\alpha)(C_y - \alpha C_x) + n\rho^2(1-\alpha^2))}{n}\right)^2 * \frac{2\gamma}{1-\alpha^2}} \tag{7}$$

$$C_x = \sum_{i=1}^{n} C_i - 1 \tag{8}$$

$$C_y = \sum_{i=1}^{n} C_i \tag{9}$$

$$C_{xx} = \sum_{i=1}^{n} C_i^2 - 1 \tag{10}$$

$$C_{xy} = \sum_{i=1}^{n} C_i - 1C_i \tag{11}$$

$$C_{yy} = \sum_{i=1}^{n} C_i^2 \tag{12}$$

It was suggested for any $t > t_0$, the OU process of C_t followed normal random variable [19]. To validate the OU process, the property of mean and variance can be estimated by the equation given below:

$$mean\ C_t = C_0 e^{-(t-t_0)\gamma} + \rho(1 - e^{(-\gamma(t-t_0))}) \tag{13}$$

$$var\ C_t = \frac{\sigma^2}{2\gamma}(1 - e^{-2(t-t_0)}) \tag{14}$$

The stochastic framework used a set of the mathematical formulation (15) - (18) for generating production phase design based on uncertain commodity price [20, 21, 22]:

$$\text{Max} \sum_{\gamma\in\Gamma}\sum_{b\in B} c_{\gamma b}x_b \tag{15}$$

$$x_b - x_{b'} \leq 0, \quad b' \in \xi_b, \quad b \in B \tag{16}$$

$$\sum_{b\in B} a_{k\gamma b}x_b \leq H_k, \quad \forall k \in K, \gamma \in \Gamma \tag{17}$$

$$x_b \in \{0,1\}, \quad b \in B \tag{18}$$

where block economic value (BEV) of block b for price scenario γ is $c_{\gamma b}$. x_b is defined as the binary variable, either 1 (if the mining block b inside of the pit) or 0 (if the mining block b outside of the pit). A set of precedence mining blocks that are necessary to be extracted before block b is ξ_b. Sum of material k in block b for price scenario γ is $a_{k\gamma b}$. The extraction rate of material k is H_k. The total number of mining blocks is B. The total number of price scenarios is Γ. Extraction rate constraint is K.

To maximize the profit, the objective function (Eq. 15) followed the rule of the precedence mining blocks (Eq. 16) and the extraction rate (Eq. 17). A stochastic method for the ultimate pit limit was eliminating the constraint of extraction rate (Eq. 17). Ultimately, a block economic value of block b for price scenario γ ($v_{\gamma b}$) is:

$$c_{\gamma b} = \left[\left(P_\gamma - r\right)g_b y - m - c\right]Q_b \tag{19}$$

$$c_{\gamma b} = \begin{cases} c_{\gamma b} & \rightarrow if\ (\left(P_\gamma - r\right)g_b y > c, block\ b\ is\ defined\ as\ coal \\ -mQ_b & \rightarrow if\ (\left(P_\gamma - r\right)g_b y \leq c, block\ b\ is\ defined\ as\ overburden \end{cases}$$

Since we used the data from the Ministry of Energy and Mineral Resources of Indonesia, Eq. 19 needs to be adjusted, as shown below:

$$c_{\gamma b}\ coal = \frac{g_b}{6322} * Qc_b * y * \left(P_\gamma - r_c\right) - (m_c * Qc_b) - (c_c * Qc_b) \tag{20}$$

$$c_{\gamma b}\ overburden = \ m_w * Qw_b \tag{21}$$

where g_b is CV of mining block b based on scenario γ (kcal), Qc_b is coal tonnage of mining block b with specific gravity equals to 1.3 ton/m3 (ton), y mining recovery (%), P_γ is coal price under scenario γ ($/ton), r_c is selling cost (2% of the coal selling price, $/ton), m_c is coal mining cost ($/ton), c_c is coal washing cost ($/ton), m_w is overburden mining cost ($/ton), and Qw_b is overburden tonnage of mining block b with specific gravity 2.1 ton/m3.

A maximum flow minimum cut algorithm can generate the optimal solution for this scenario [3, 23, 24]. **FIGURE 2a** shows a fundamental graph structure incorporating various price scenarios. The graph is equipped with a source node, a node of mining block b under each price scenario γ, and a sink node. For price scenario γ, each mining block b holds the same probability of an economic value $c_{\gamma b}$. Therefore, if $c_{\gamma b} > 0$, the node is linked to the source node; otherwise, the node is linked to the sink node for $c_{\gamma b} \leq 0$. The capacity of an arc linked to the source node is a positive-valued node defined as $c_{\gamma b}$. Likewise, the capacity of an arc linked to a sink node is a negative-valued node defined as $\left|c_{\gamma b}\right|$. Also, an infinite (∞) capacity arc ensures the constraint to extract precedence mining block. This arc two from a mining block b for price scenario γ to a set of nodes in ξ_b. According to Dimitrakopoulos et al. [25] these infinite (∞) capacity arcs played a role, while generating an optimal solution, an existing mining block b in Γ number of price scenarios, stayed on the same side of cut in all price scenarios, and finally created a valid optimal pit.

A particular block is always on the same side of the cut in the different simulations by maintaining the bidirectional arcs across the simulations. The bidirectional arcs allowed the merging of the nodes from the different simulations into a single node. The process reduced the number of nodes within the graph and ultimately made the process less computationally demanding. The fact that a mining block b stays on the same side of cut in all α scenarios, the nodes can be updated as $\left[\tilde{c}_b = \sum_\gamma^\Gamma c_{\gamma b}\ ; if\ c_{\gamma b} > 0\right]$, and the arc capacity of the single merged node to the sink node is updated as $\left[\hat{c}_b = \sum_\gamma^\Gamma c_{\gamma b}\ ; if\ c_{\gamma b} \leq 0\right]$. **FIGURE 2b** shows the merged nodes of the two simulations in **FIGURE 2a**.

The price simulation produced multiple simulated prices over several time periods t from 1 to T. Assuming the time period t, the coal selling price P_γ could be substituted with $P_{\gamma t}$ in Eq. 19. If d is the discount rate, the discounted value of block b over T number of time periods is represented by c_{bt} as stated below:

$$\tilde{c}_{bt} = \sum_\gamma^\Gamma \frac{c_{bt}}{(1+d)^t}\ if\ c_{\gamma b} > 0 \tag{22}$$

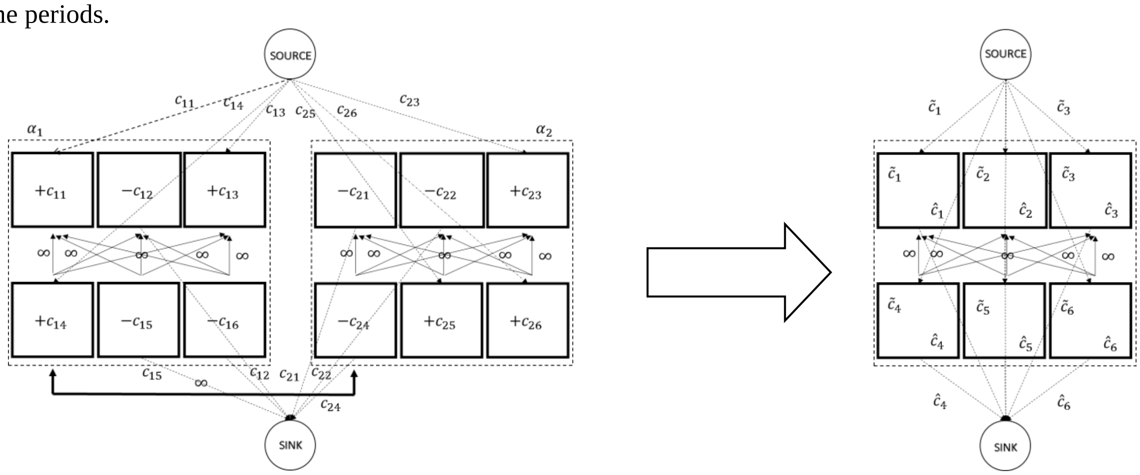
$$\hat{c}_{bt} = \sum_{\gamma}^{\Gamma} \frac{c_{bt}}{(1+d)^t} \; if \; c_{\gamma b} \leq 0 \tag{23}$$

The discounted value of block b over a T number of time periods is represented by c_{bt} as stated below:

$$\tilde{c}_{bt} \; coal = \sum_{t}^{T} \frac{c_{bt} \; coal}{(1+d)^t} \tag{24}$$

$$\hat{c}_{bt} \; overburden = \sum_{t}^{T} \frac{c_{bt} \; overburden}{(1+d)^t} \tag{25}$$

Infinite-capacity arcs (∞) assisted not only to ensure precedence blocks mined but also to ensure the mining block b within the valid pit over time periods T only mined once over a T number of time periods. The next step was same as the undiscounted BEV. **FIGURE 3** shows the implementation of the discounted block economic value over several time periods.



(a) (b)

FIGURE 2. Constructed graph of multiple coal body realizations (a) and Merged graph of two simulated models.

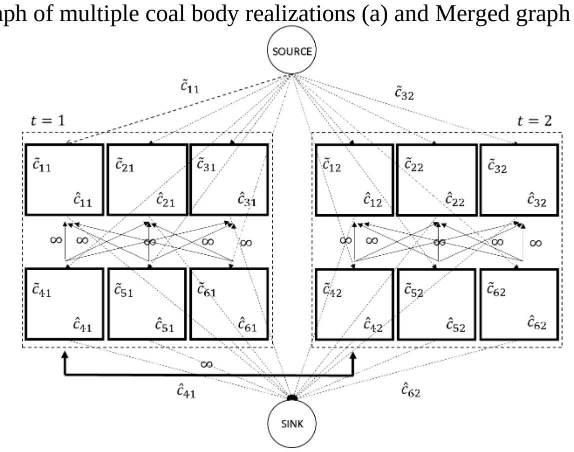


FIGURE 3. The integration of the modified graph with discounted values of mining block.

Extraction capacity constraints were not considered in the stochastic algorithm mentioned above. The integration of the extraction constraints was necessary to create reasonable and valid phase-design [25, 26]. Implementing the maximum flow minimum cut algorithm only produced an invalid solution for the problem [4, 5]. Extraction capacity constraints were added into phase design and Lagrangian parameter (λ) by Tachefine and Soumis [27]. Considering H is a set of extraction constraints, a_{kb} is the sum of material k inside of block b, and H_k is the extraction capacity of material k, the Eq. (15) - (17) can be updated with a new extraction constraint by combining all uncertainty scenarios mentioned earlier:

$$\sum_{b \in B} a_{k\gamma b} x_b \leq H_k, k \in K \; and \; \gamma \in \Gamma \tag{26}$$

the new Lagrangian formula;

Maximize $Z(\lambda) = \sum_{b \in B} \sum_{t \in T} \lambda \left[\sum_{\gamma \in \Gamma} c_{\gamma bt}\right] x_{bt}$ (27)

Subject to $x_{bt} - x_{b't} \leq 0, \forall t, b' \epsilon\, \xi_b, b \in B$ (block b' overlies block b) (28)

$x_{bt} \in \{0,1\}, b \in B$ and $\sum_{t \in T} x_{bt} \leq 1$ (29)

and $\lambda = \begin{cases} \lambda, & \sum_{\gamma \in \Gamma} c_{\gamma bt} > 0 \\ 1, & otherwise \end{cases}$ (30)

where λ is curbed with (0,1], and λ changes to $\lambda = \lambda + \nabla\lambda$ in each iteration. The iteration updated the arc capacity of from the source node to positive discounted value nodes, produced a consistent size of phase-designs. A smaller or higher step size or $\nabla\lambda$ impacted differently on the generating phase-design. A smaller value of the step size created a computationally demanding calculation. However, a higher value of the step size generated unacceptable gaps [4]. Assuming ϵ is a very small positive number, a λ will be updated to a monotonically decreasing function as stated below:

$$\nabla\lambda = \left[\sum_{k \in K} \sum_{\gamma \in \Gamma} w_{k\gamma} \left(\frac{H_k - \sum_{b \in B} a_{k\gamma b}}{H_k}\right)\right] \epsilon \quad (31)$$

The flows of the stochastic framework solved with a push-relabel algorithm to develop an ultimate pit limit and production phase-design stated below:

1. For each node and scenario, calculate the undiscounted BEV using Eq. (20) and (21).
2. Merge all the nodes with calculated BEV over all the simulations into a single node and compute the discounted BEV over time periods using Eq. (24) and (25).
3. Define a variable λ so that 0 < λ < 1.
4. Set initial λ = 0.
5. For all production capacity constraints k for scenario γ, set $\delta_{k\gamma} = 0$ when λ = 0.
6. Increment λ by a small value $\nabla\lambda$, such that $\lambda = \lambda + \nabla\lambda$.
7. Calculate $Z(\lambda)$ in Equation (27).
8. If $[\![\delta_{k\gamma} = \sum_{b \in B} a_{k\gamma b} x_b] \leq H_k \forall\gamma, k]$ from $Z(\lambda)$ in Equation (27), update $\lambda = \lambda + \nabla\lambda$.
9. Go to step 7.
10. End.

RESULTS AND DISCUSSION

The coal reference price of Ministry of Energy and Mineral Resources of Indonesia uses the calorific value of 6322 kcal/kg GAR as a benchmark (other parameters, e.g. total moisture, total sulfur, and ash are not the scope of this research). The research used the coal reference price from January 2009 to January 2018. The coal price scenarios were needed to be simulated for 50 times over the next ten years. The mean-reversion process started with the calibrating the parameters in Eq. 3. Three calibrated parameters; ρ (mean-reversion level), σ (volatility), and γ (mean-reversion speed). For the validation model, 10% of the historical coal price data were excluded. Next, these data were used for a paired-sample t-test statistics to know whether the model and the actual data have the same distribution. From Eq. (4) – (12), the parameters of the mean-reversion process could be derived. To improve the calibration process of γ, the jackknife method was integrated into the process. The value of C_0 was $ 101.69. From Eq. (4) – (12) resulted parameters of OU mean-reversion for γ = 0.2521, ρ = 93.1561, and σ = 15.9592. It was seen here that the coal price data had a very slow speed of reverting price (0.2521). These parameters were used along with the Monte Carlo simulation by setting up several simulations equal to 50 and the number of steps (months) equal to 120. The result of the Monte Carlo simulation is shown in **FIGURE 4**.

The validation was performed by calculating the expected mean and variance vs. the mean and variance of the price simulation using Eq. (13) and (14). The results of the first and second momentum can be seen in **FIGURE 5**. In this case, a paired-sample t-test statistic (two-tailed test statistic) was also performed to see whether the model and actual data have the same distribution. The null hypothesis was set to that there was no significant difference mean between the model and the actual prices, and the alternative hypothesis was vice versa. By using 10% of the available historical coal price with 5% significance level, there was no significant difference between the mean of the model and the actual price. **TABLE 2** shows the statistic result of 10% of the historical coal prices compared to the price simulations. With that was -2.12, it fell in the nonrejection region for a significance level of 0.05. The result implied the mean-reversion process model could be used to simulate future coal prices.

The stochastic model used jointly multiple realizations of the coal price of a mining block to simulate block economic value (BEV) using Equation (20-21) - (24-25). It produced 113,050 blocks of economic value over 50 simulations each year. Afterward, a single graph was constructed (**FIGURE 3**). By applying the slope constraint, the block number decreased from 113,050 to 93,306 blocks. The total blocks were used to generate production phase-designs. Production phase-design considered Eq. (31) to generate a solution. The function controlled the size of the individual phase-design by constraining it with mining production rate per year of 45 million tonnes of material (including overburden and coal). The proposed method produced ten production phase-designs within the ultimate pit limit.

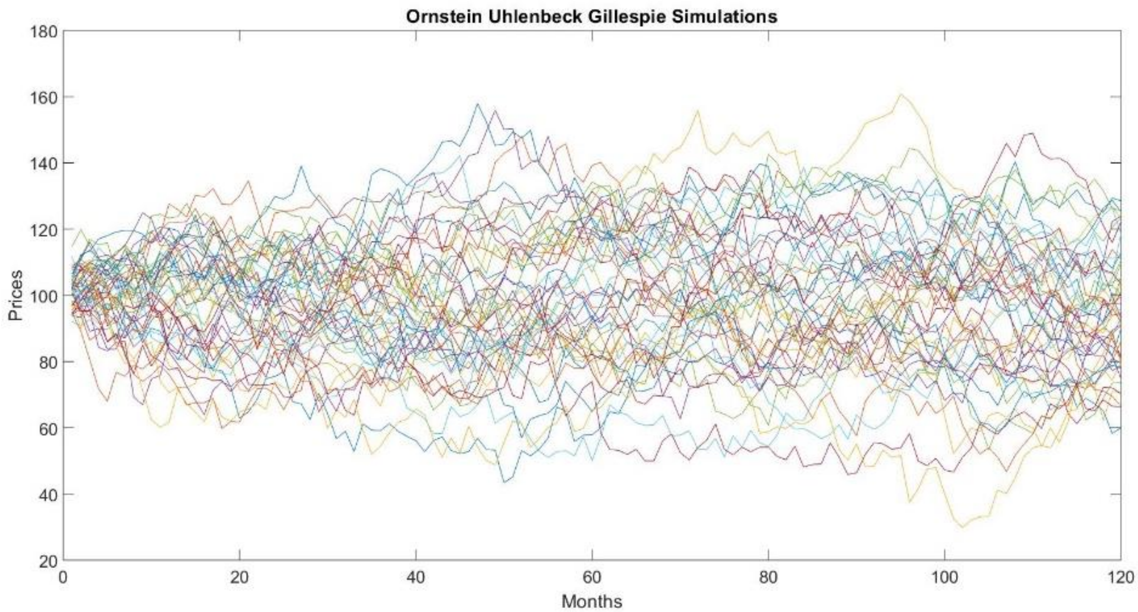


FIGURE 4. The Monte Carlo simulation from the OU mean-reversion process.

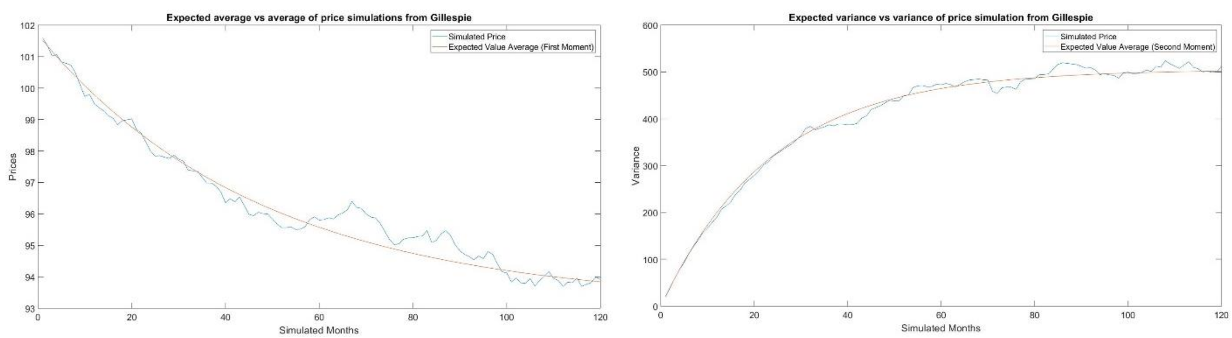


FIGURE 5. The expected average vs. the average of the price simulations (left) and the expected variance vs. the variance of the price simulations (right).

Although there were two gaps between phase-design number 2 and 4 resulting small phase-design in number 5, the total of removed material from the pit was close to the 10-year production target (450 million tons of material), and the life of mine was close to 10 years. The total blocks remain inside the ultimate pit limit is 67,026. To generate the discounted cash flow of the phase-design over the 10-year life of mine, BEV calculation of each year was used. The undiscounted BEV of each year from Equation (24) – (25) was needed to be multiplied by a discounted factor of 9.5%. Since the life of each phase-design in each year was not round 1, to calculate the discounted cash flow, prorating of the cash flow from each year was performed to get 1-year cash flow with the specified discounted rate. **TABLE 3** shows the risk profile of the phase-designs with minimum, maximum, and average cash flow.

TABLE 2. The result of a paired-sample t-test with a significance level (α) = 0.05

No	Actual	Model	Statistical Properties	
1	86.23	90.96		
2	83.32	86.53		
3	81.90	84.16	t statistic	-2.12
4	82.51	87.32		
5	83.81	90.64		
6	75.46	80.96		
7	78.95	88.33		
8	83.97	89.88		
9	92.03	90.87		
10	93.99	97.72	critical value	2.18
11	94.80	96.54		
12	94.04	88.22		
13	95.54	89.55		

TABLE 3. The production phase designs and risk profile in a stochastic framework with monotonous decrease λ

Phase Number	Coal Block	OB Block	CV (kcal/kg)			Quantity of Material (Ton)	Life (Years)
			Minimum	Average	Maximum		
1	4543	2637	5074.52	5854.58	6570.67	45,774,400	1.02
2	5993	4228	5131.98	5895.55	6597.47	66,678,800	1.48
3	3102	2400	5171.51	5912.19	6697.18	36,290,400	0.81
4	5155	4479	5125.91	6139.86	6656.57	64,429,600	1.43
5	1019	914	5182.19	6060.94	6619.51	12,976,400	0.29
6	4001	3621	5112.29	6004.31	6610.94	51,221,600	1.14
7	2958	2854	5158.89	6055.53	6620.83	39,355,200	0.87
8	3551	3858	5216.68	6076.96	6636.91	50,872,400	1.13
9	2745	2864	5178.12	6104.26	6753.98	38,331,600	0.85
10	2066	4038	5151.31	6092.46	6766.98	44,662,400	0.99
Ultimate Pit	35133	31893	5150.34	6019.66	6653.10	450,592,800	10.01

Risk Profile (Discounted Case Flow)

Phase	Minimum (US$ Million)	Maximum (US$ Million)	Average (US$ Million)
1	1,615.90	2,204.10	1,860.10
2	3,601.90	4,538.30	3,998.80
3	3,743.40	5,029.90	4,443.80
4	3,426.00	4,416.00	3,892.70
5	3,797.30	5,570.70	4,872.00
6	4,416.00	6,769.40	5,452.50
7	4,109.70	6,747.40	5,452.10
8	4,832.30	7,723.50	6,684.60
9	3,968.70	5,823.50	4,966.80
10	4,441.10	6,441.30	5,517.80
Ultimate Pit	37,696.40	55,264.10	47,141.20

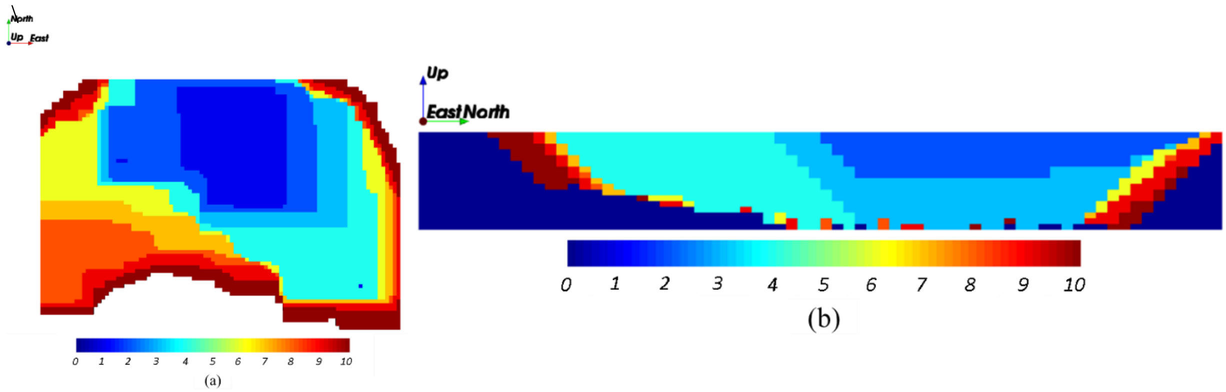


FIGURE 6. North-East (a) and North-South (b) views of production phase-design in the stochastic method (0 -10 is indicated year of production).

Comparison between stochastic and conventional (deterministic) approach is described here. The stochastic approach integrates multiple coal price simulation while the conventional approach uses only a single coal price as the input of the process. The single coal price is obtained from averaging all the price scenarios within a period. For a valid comparison, all parameters remain the same. The algorithm in a conventional approach produces 10 phase-designs within ultimate pit limit. This is similar to the stochastic model. From **TABLE 4**, it shows that the number of blocks (coal + overburden) and NPV are smaller if it is compared to the stochastic model. The blocks in the deterministic model equal to 66,946 or 0.2 % smaller than the stochastic model. The difference between the stochastic and deterministic model is not big since the monotonous decrease λ algorithm that used in this paper will stop the iteration only if the production each phase equal or one iteration greater than the target. This also will make the gap problem appear since the production of the consecutive pushback sometimes is not really close to the specified target.

TABLE 4. The production phase design and risk profile in a stochastic framework with the deterministic model.

Phase Number	Coal Block	OB Block	CV (kcal/kg)			Quantity of Material (Ton)	Life (Years)
			Minimum	Average	Maximum		
1	4502	2660	5073.52	5854.58	6570.67	45,989,600	1.02
2	5946	4144	5121.98	5895.55	6597.47	65,798,800	1.46
3	3269	2348	5161.51	5912.19	6697.18	36,564,000	0.81
4	5459	4199	5125.91	6139.86	6656.57	63,106,400	1.40
5	1084	841	5182.19	6060.94	6619.51	12,742,800	0.28
6	4213	3465	5112.29	6004.31	6610.94	50,536,800	1.12
7	3037	2715	5158.89	6055.53	6620.83	39,224,800	0.87
8	3671	3732	5116.68	6076.96	6636.91	50,482,000	1.12
9	2289	3316	5078.12	6104.26	6753.98	39,733,200	0.88
10	1622	4434	5101.31	6092.46	6766.98	45,873,600	1.02
Ultimate Pit	35092	31854	5123.24	6019.66	6653.10	450,592,000	10.00

Risk Profile (Discounted Case Flow)

Phase	Minimum (US$ Million)	Maximum (US$ Million)	Average (US$ Million)
1	1,517.90	2,134.10	1,826.00
2	3,401.30	4,514.80	3,958.05

3	3,711.10	5,020.60	4,365.85
4	3,415.10	4,311.50	3,873.30
5	3,790.20	5,511.60	4,650.90
6	4,138.20	6,622.90	5,440.55
7	4,011.10	6,713.10	5,362.10
8	4,711.10	7,620.70	6,165.90
9	3,818.10	5,726.00	4,772.05
10	4,321.10	6,321.10	5,341.10
Ultimate Pit	36,835.20	54,496.40	45,755.80

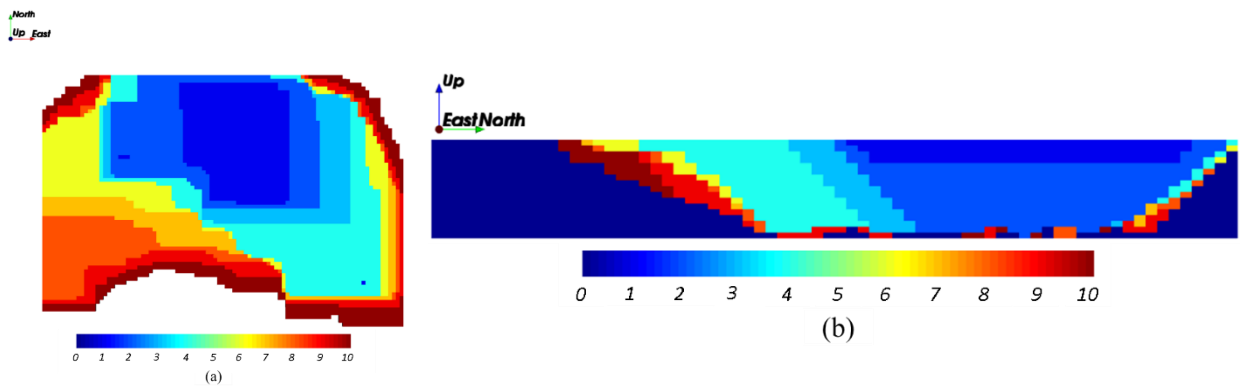


FIGURE 7. North-East (a) and North-South (b) views of the deterministic production phase-design (0 -10 is indicated year of production).

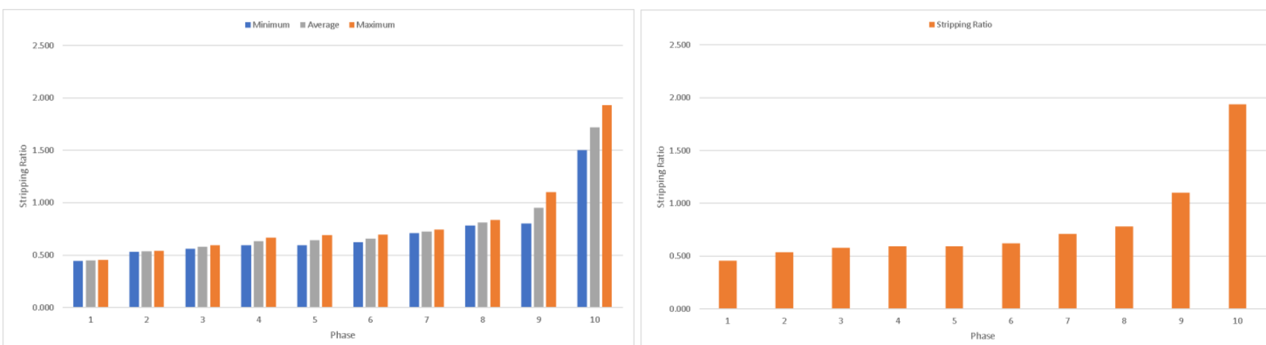


FIGURE 8. The stripping ratio profile of the in the stochastic method (left) dan conventional method (right).

CONCLUSIONS

This paper discussed generating phase design and ultimate pit limit by incorporating uncertain coal price using the maximum flow minimum cut algorithm. Multiple commodity price scenarios were integrated into the stochastic framework, which is different from the conventional method that used a single price over the life of mine. The proposed method revealed that incorporating uncertain commodity price into the minimum cut algorithm model was easy to construct and resolve. This paper also implemented a method for simulating coal prices. They were obtained by estimating the long term mean and mean reversion speed of the historical coal price using the Ornstein-Uhlenbeck (OU) algorithm. The proposed method promoted the incorporation of uncertain coal prices into the optimization framework. An application of Indonesian coal mine was used in this research and resulted in 10 production phase-design, a larger pit (0.2%), and critically, a higher net present value (42 %) as compared to the conventional method. The overall stripping ratio of the conventional model was slightly bigger than the stochastic model, which is 1.62.

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REFERENCES

[1] M. Godoy and R. Dimitrakopoulos, "A risk quantification framework for strategic mine planning: Method and application," *Journal of Mining Science,* vol. 47, no. 2, pp. 235-246, 2011.

[2] E. Moosavi and J. Gholamnejad, "Long-term production scheduling modeling for the open pit mines considering tonnage uncertainty via indicator kriging," *Journal of Mining Science,* vol. 51, no. 6, pp. 1226-1234, 2015.

[3] A. Goldberg and R. Tarjan, "A new approach to the maximum-flow problem," *Journal of the ACM,* vol. 35, no. 4, pp. 921-940, 1988.

[4] J. Picard, "Maximal Closure of a Graph and Applications to Combinatorial Problems," *Management Science,* vol. 22, no. 11, pp. 1268-1272, 1976.

[5] S. Chatterjee, M. Sethi and M. Asad, "Production phase and ultimate pit limit design under commodity price uncertainty," *European Journal of Operational Research,* vol. 248, no. 2, pp. 658-667, 2016.

[6] O. Vasicek, "An equilibrium characterization of the term structure," *Journal of Financial Economics,* vol. 5, no. 2, pp. 177-188, 1977.

[7] J. Cox, J. Ingersoll and S. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica,* vol. 53, no. 2, p. 385, 1985.

[8] R. Pindyck, "The Long-Run Evolutions of Energy Prices," *The Energy Journal,* vol. 20, no. 2, 1999.

[9] Q. Sun, W. Xu and W. Xiao, "An empirical estimation for mean-reverting coal prices with long memory," *Economic Modelling,* vol. 33, pp. 174-181, 2013.

[10] A. A. Charles and O. Darné, "VARIANCE-RATIO TESTS OF RANDOM WALK: AN OVERVIEW," *Journal of Economic Surveys,* vol. 23, no. 3, pp. 503-527, 2009.

[11] J. Lucia and E. Schwartz, Electricity prices and the power derivatives, Los Angeles: UCLA Center for International Business Education and Research, 2000.

[12] N. Meade, "Oil prices — Brownian motion or mean reversion? A study using a one year ahead density forecast criterion," *Energy Economics,* vol. 32, no. 6, pp. 1485-1498, 2010.

[13] S. Schlueter, "A long-term/short-term model for daily electricity prices with dynamic volatility," *Energy Economics,* vol. 32, no. 5, pp. 1074-1081, 2010.

[14] E. Schwartz, "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging," *The Journal of Finance,* vol. 52, no. 3, p. 923, 1997.

[15] L. Valdivieso, W. Schoutens and F. Tuerlinckx, "Maximum likelihood estimation in processes of Ornstein-Uhlenbeck type," *Statistical Inference for Stochastic Processes,* vol. 12, no. 1, pp. 1-19, 2008.

[16] W. Smith, "Commoditymodels.files.wordpress.com," 2010. [Online]. Available: https://commoditymodels.files.wordpress.com/2010/02/estimating-the-parameters-of-a-mean-reverting-ornstein-uhlenbeck-process1.pdf.. [Accessed 20 October 2017].

[17] J. Yu, "Bias in the estimation of the mean reversion parameter in continuous time models," *Journal of Econometrics,* vol. 169, no. 1, pp. 114-122, 2012.

[18] P. Phillips and J. Yu, "Jackknifing Bond Option Prices," *Review of Financial Studies,* vol. 18, no. 2, pp. 707-742, 2005.

[19] D. Gillespie, "Exact numerical simulation of the Ornstein-Uhlenbeck process and its integral," *Physical Review E,* vol. 54, no. 2, pp. 2084-2091, 1996.

[20] S. Ramazan, "The new Fundamental Tree Algorithm for production scheduling of open pit mines," *European Journal of Operational Research,* vol. 2007, no. 2, pp. 1153-1166, 2007.

[21] R. Dimitrakopoulos and S. Ramazan, "Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions," *Mining Technology,* vol. 117, no. 4, pp. 155-160, 2008.

[22] S. Ramazan and R. Dimitrakopoulos, "Production scheduling with uncertain supply: a new solution to the open pit mining problem," *Optimization and Engineering,* vol. 14, no. 2, pp. 361-380, 2012.

[23] D. Hochbaum, "A new?old algorithm for minimum-cut and maximum-flow in closure graphs," *Networks,* vol. 37, no. 4, pp. 171-193, 2001.

[24] D. Hochbaum and A. Chen, "Performance Analysis and Best Implementations of Old and New Algorithms for the Open-Pit Mining Problem," *Operations Research,* vol. 48, no. 6, pp. 894-914, 2000.

[25] R. Dimitrakopoulos, Advances in orebody modelling and strategic mine planning, Carlton South: AusIMM, 2010.

[26] A. Goldberg and R. Tarjan, "A new approach to the maximum-flow problem," *Journal of the ACM,* vol. 35, no. 4, pp. 921-940, 1988.

[27] B. Tachefine and F. Soumis, "Maximal closure on a graph with resource constraints," *Computers & Operations Research,* vol. 24, no. 10, pp. 981-990, 1997.