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## International Conference of

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Jember,Indonesia
25-26 November 2017

Volume: 1008-2018
ISSN: 17426588


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To cite this article: 2018 J. Phys.: Conf. Ser. 1008011001

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# The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT) 

Dafik<br>Editor in Chief of ICCGANTs Publication, University of Jember, Jember, Indonesia<br>E-mail: d.dafik@unej.ac.id

## Preface

It is with my great pleasure and honor to organize the First International Conference on Combinatorics, Graph Theory and Network Topology which is held from 25-26 November 2017 in the University of Jember, East Java, Indonesia and present a conference proceeding index by Scopus. It is the first international conference organized by CGANT Research Group University of Jember in cooperation with Indonesian Combinatorics Society (INACOBMS). The conference is held to welcome participants from many countries, with broad and diverse research interests of mathematics especially combinatorical study. The mission is to become an annual international forum in the future, where, civil society organization and representative, research students, academics and researchers, scholars, scientist, teachers and practitioners from all over the world could meet in and exchange an idea to share and to discuss theoretical and practical knowledge about mathematics and its applications. The aim of the first conference is to present and discuss the latest research that contributes to the sharing of new theoretical, methodological and empirical knowledge and a better understanding in the area mathematics, application of mathematics as well as mathematics education.

The themes of this conference are as follows: (1) Connection of distance to other graph properties, (2) Degree/diameter problem, (3) Distance-transitive and distance-regular graphs, (4) Metric dimension and related parameters, (5) Cages and eccentric graphs, (6) Cycles and factors in graphs, (7) Large graphs and digraphs, (8) Spectral Techniques in graph theory, (9) Ramsey numbers, (10) Dimensions of graphs, (11) Communication networks, (12) Coding theory, (13) Cryptography, (14) Rainbow connection, (15) Graph labelings and coloring, (16). Applications of graph theory

The topics are not limited to the above themes but they also include the mathematical application research of interest in general including mathematics education, such as:(1) Applied Mathematics and Modelling, (2) Applied Physics: Mathematical Physics, Biological Physics, Chemistry Physics,(3) Applied Engineering: Mathematical Engineering, Mechanical engineering, Informatics Engineering, Civil Engineering,(4) Statistics and Its Application,(5) Pure Mathematics (Analysis, Algebra and Geometry),(6) Mathematics Education, (7) Literacy of Mathematics,(8) The Use of ICT Based Media In Mathematics Teaching and Learning,(9) Technological, Pedagogical, Content Knowledge for Teaching Mathematics, (10) Students Higher Order Thinking Skill of Mathematics, (11) Contextual Teaching and Realistic Mathematics, (12) Science, Technology, Engineering, and Mathematics Approach, (13) Local Wisdom Based

Education: Ethnomathematics, (14) Showcase of Teaching and Learning of Mathematics, (16) The 21st Century Skills: The Integration of 4C Skill in Teaching Math.

The participants of this ICCGANT 2017 conference were 200 people consisting research students, academics and researchers, scholars, scientist, teachers and practitioners from many countries. The selected papers to be publish of Journal of Physics: Conference Series are 80 papers. On behalf of the organizing committee, finally we gratefully acknowledge the support from the University of Jember of this conference. We would also like to extend our thanks to all lovely participants who are joining this unforgettable and valuable event.

Prof. Drs. Dafik, M.Sc., Ph.D.

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## Continuous connection of two adjacent pipe parts defined by line, bézier and hermit center curves

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# Continuous connection of two adjacent pipe parts defined by line, bézier and hermit center curves 

$K^{\prime}{ }^{1}{ }^{1}$, Antonius Cahyo Prihandoko ${ }^{1}$<br>${ }^{1}$ Department of Mathematics, University of Jember, Jember, Indonesia<br>E-mail: kusno.fmipa@unej.ac.id


#### Abstract

A shape of pipe part consist of the longitudinal and cross section boundary curve. Also, it can be defined by only its center curve of the pipe. To build a complete pipe, we can connect continually the pipe parts. This paper discuss about continuous connection of two adjacent pipe parts that are defined by line, Bézier and Hermit curves. The method is as follows, we join continually two adjacent center curves of the pipe parts and then by using the same formulae, we define its coincidence cross section boundary curves of the pipes. Finally, the joining of the longitudinal section boundary curves of the pipe parts can be done. The results of this study show that we can evaluate the joint continuity between two adjacent pipe parts defined by line, Bézier and Hermit curves, using the various forms of its center and cross-longitudinal section boundary curves.


## 1. Introduction

There are many introduced methods for modeling a shape of pipe. We can use a single-valued tubular patches and a cyclide to model radius and pipe design of tubular geometry [7,8]. By using the geometric continuity condition for algebraic surface we can unify some pieces of pipes at common vertex [1]. By joining two cylinders of revolution with axes in a common plane and diff erent radii we will find a circular surfaces [4]. Also, the shape of pipe can be evaluated by geometric properties of canal surfaces in $E^{3}[6]$. The physical model of transitional pipeline parts can be made of materials that cannot be wrinkled or stretched [5]. Then, a modular Pipe-Z parametric design system will give the results for a trefoil, a gure-eight knot and a pentafoil [9]. Diff erent from the earlier methods, this paper discusses about constructing the whole pipe using small parts connection of the pipes. In this discussion, we calculate the continuous connection of two adjacent pipe parts defined by line, Bézier and Hermit curves.

This paper is organized in the following sections. In the first section, we talk about the formulation of pipe parts that depend on its cross, longitudinal and center curves. In the second, we evaluate the continuous connection of two adjacent pipe parts of the line, Bézier and Hermit center curves. Finally, the results will be summarized in the conclusion section.

## 2. Pipe Parts Formulation

Consider a regular curve $\Gamma(u)$ continuous on the interval $0 \leq u \leq 1$ that can be diff erentiated twice. Also it can be expressed as a function of the natural parameter $\Gamma(s)$. The tangent unit vector $\mathbf{t}$ and the normal unit vector $\mathbf{n}$ are orthogonal in the form (Figure 1a)

$$
\begin{align*}
& \mathbf{t}=\frac{d \boldsymbol{\Gamma}}{d s}=\dot{\mathbf{\Gamma}}=\frac{\boldsymbol{\Gamma}^{\prime}}{\left|\boldsymbol{\Gamma}^{\prime}\right|}  \tag{1a}\\
& \mathbf{n}=-\frac{\mathbf{k}}{|\mathbf{k}|} \tag{1b}
\end{align*}
$$

and

$$
\begin{equation*}
\mathbf{k}=\frac{d \mathbf{t}}{d s}=\dot{\mathbf{t}}=\frac{\mathbf{t}^{\prime}}{\left|\Gamma^{\prime}\right|} \tag{1c}
\end{equation*}
$$

Using the cross product operation of the both vectors $\mathbf{t}$ and $\mathbf{n}$, we can define the unit binormal vector $\mathbf{b}=\mathbf{t} \wedge \mathbf{n}$ such that the triplet $[\mathbf{t}, \mathbf{n}, \mathbf{b}]$ form the Frenet frame of curve [3].

Consider along the direction of parameter $u$ each points of the curve $\Gamma(u)$ as the center points of the defined circles of radius $\gamma(u, v)$ in the normal plane $[\mathbf{b}, \mathbf{n}]$ of the curve $\Gamma(u)$ that are orthogonal to unit tangent vector $\mathbf{t}$. Using the parametric tubular surface formulae of the curve $\Gamma(u)$, we can define a part of tubular pipe in the form [2]

$$
\begin{equation*}
\mathbf{T}(u, v)=\Gamma(u)+\Upsilon(u, v) \cdot[\cos (\varphi) \mathbf{b}+\sin (\varphi) \mathbf{n}] \tag{2}
\end{equation*}
$$

with the real function $\gamma(u, v)=\rho(u) \cdot r(v)$ expresses the radius of the pipe, $\varphi=2 \pi v$ and $0 \leq u, v \leq 1$. In this case, the real function $\rho(u)$ characterizes the inflate-deflate surfaces form of the pipe patches along its center curve, meanwhile $r(v)$ defines the cross-section curve form of the pipe. To facilitate the creation of pipe part, we determine the real function $\rho(u)$ and $r(v)$ from the Bézier, Hermit and trigonometric curves as follows

$$
\begin{align*}
& \rho_{1}(u)=P_{o}(1-u)^{3}+3 P_{1}(1-u)^{2} u+3 P_{2}(1-u) u^{2}+P_{3} u^{3}  \tag{3a}\\
& \rho_{2}(u)=\rho_{2}(0) H_{1}(u)+\rho_{2}(1) H_{2}(u)+\rho_{2}^{u}(0) H_{3}(u)+\rho_{2}^{u}(1) H_{4}(u)  \tag{3b}\\
& \rho_{3}(u)=a+b \cos (u)+c \sin (u)  \tag{3c}\\
& r_{I}(v)=a \cdot \cos (n \cdot \varphi) \pm b \cdot \sin (n \cdot \varphi) \tag{4a}
\end{align*}
$$

where

$$
\begin{array}{ll}
H_{1}(u)=2 u^{3}-3 u^{2}+1 ; & H_{2}(u)=2 u^{3}+3 u^{2} ; \\
H_{3}(u)=u^{3}-2 u^{2}+u ; & H_{4}(u)=u^{3}-u^{2}
\end{array}
$$

with $a, b, c$ real constants, $\varphi=2 \pi \cdot v$ and $n$ is the number of defined rose leafs, $0 \leq u \leq 1$ and $0 \leq v$ $\leq 1$. In addition, we define the cross section curve of pipe using unify $n$ curves of the different circle parts to the same origin O in the form

$$
\begin{equation*}
r_{2}(v)=r_{o} \cdot\left[\cos \left((2 i+1) \cdot \frac{\pi}{n}-\frac{\varphi}{n}\right)\right] \pm \sqrt{\left[r_{o}^{2} \cdot \cos ^{2}\left((2 i+1) \cdot \frac{\pi}{n}-\frac{\varphi}{n}\right)-\left(r_{o}^{2}-\tau^{2}\right)\right]} \tag{4b}
\end{equation*}
$$

for $i=0,1, \ldots, n-1$ and $r_{0}$ as a ray of polar form, $\varphi=2 \pi v$ with $0 \leq v \leq 1$. The first, we determine $\Gamma(u)$ as a line curve of equation

$$
\begin{equation*}
\mathbf{L}(u)=\mathbf{c}+\lambda u . \mathbf{l} \tag{5}
\end{equation*}
$$

with $\mathbf{c}$ constant vector, $\mathbf{I}$ unit direction vector, $\lambda$ positive real constant and $0 \leq u \leq 1$. Because of the $\mathbf{L}(u)$ line, we must change the circles of radius $\gamma(u, v)$ in the normal plane $[\mathbf{b}, \mathbf{n}]$ in equation (2) become in plane $\left[\mathbf{v}_{1}, \mathbf{v}_{2}\right]$ with $\mathbf{v}_{1}$ and $\mathbf{v}_{2}$ any unit constant vectors such that $\mathbf{l}=\mathbf{v}_{1} \wedge \mathbf{v}_{2}$. The second, we define the center curve $\Gamma(u)$ in the form cube Bézier curve $\mathbf{B}(u)$ and Hermit curve $\mathbf{H}(u)$ with

$$
\begin{align*}
& \mathbf{B}(u)=\mathbf{P}_{o}(1-u)^{3}+3 \mathbf{P}_{1}(1-u)^{2} u+\mathbf{P}_{2}(1-u) u^{2}+\mathbf{P}_{3} u^{3}  \tag{6}\\
& \mathbf{H}(u)=\mathbf{H}_{0} H_{1}(u)+\mathbf{H}_{1} H_{2}(u)+\mathbf{H}_{o}^{u} H_{3}(u)+\mathbf{H}_{1}^{u} H_{4}(u) . \tag{7}
\end{align*}
$$

The vectors $\mathbf{P}_{o}, \mathbf{P}_{1}, \mathbf{P}_{2}$ and $\mathbf{P}_{3}$ are the control points of the Bézier curve $\mathbf{B}(u), \mathbf{H}_{o}=\mathbf{H}(0)$, $\mathbf{H}_{1}=\mathbf{H}(1), \mathbf{H}_{o}^{u}=\mathbf{H}^{\prime}(0), \mathbf{H}_{1}^{u}=\mathbf{H}^{\prime}(1)$ and $0 \leq u \leq 1$.

The implementation of formulae (2-7) can be demonstrated as follows. Let $\mathbf{v}_{1}=\langle 0,1,0\rangle, \mathbf{v}_{2}=$ $\langle 0,0,1\rangle$ and $\mathbf{L}(u)=\langle-15,0,0\rangle+25 . u .\langle 1,0,0\rangle$. When we decide $\rho(u)=2+\cos (2 \pi u)-\sin (2 \pi u)$ and $r(v)=1$ such that $\Upsilon(u, v)=2+\cos (2 \pi u)-\sin (2 \pi u)$, equation (2) will show Figure 1a. If we determine $\rho(u)=1.5 ; r(v)=-2-\cos (4 \pi v)-\cos (8 \pi v)$ and $\rho(u)=2(1-u)^{3}+12(1-u)^{2} . u+3$ $(1-u) \cdot u^{2}+2 u^{3} ; r(v)=-2-\cos (4 \pi v)-\cos (8 \pi v)$, we will find Figure 1 b and 1 c , respectively. Figure 1d and 1e show the graph of equation (2) and (6) with control points [ $\mathbf{P}_{o}=\langle-22,-18,15\rangle$, $\left.\mathbf{P}_{1}=\langle 8,-2,-50\rangle, \mathbf{P}_{2}=\langle 18,3,16\rangle, \mathbf{P}_{3}=\langle 25,-14,10\rangle\right], \rho(u)=2(1-u)^{3}+6(1-u)^{2} \cdot u+3(1-u) \cdot u^{2}$ $+2 u^{3} ; r(v)=4$ and $\rho(u)=2\left(2 u^{3}-3 u^{2}+1\right)+2\left(2 u^{3}+3 u^{2}\right)+8\left(=u^{3}-2 u^{2}+u\right)+10\left(u^{3}-u^{2}\right) ; r(v)=$ 2. Base on the equations (1-7), then we discuss about the continuous connection of two adjacent pipe parts defined by line, Bézier and Hermit curves


Figure 1. Pipe parts of the line center curve (1a,b,c) and the cubic Bézier center curve (1d,e)

## 3. Connection of Two Adjacent Pipe Parts

Consider two parametric tubular surface pieces $\mathbf{T}_{1}(u, v)$ and $\mathbf{T}_{2}(u, v)$ of equation (2). Its center curve and its cross-longitudinal boundary curves are respectively $\left[\mathbf{L}_{1}(u), \mathbf{L}_{2}(u)\right]$ of line curve (5) and $\left[\Upsilon_{1}(u, v), \Upsilon_{2}(u, v)\right]$ of equations (3-4) in the same direction condition. Because of the center curves of degree one, we can join the both tubular surfaces using tangential continuity, that is $\mathbf{T}_{\mathrm{L} 1}(1, v)=\mathbf{T}_{\mathrm{L} 2}(0, v)$ and $\mathbf{T}_{\mathrm{L} 1}{ }^{\mathrm{u}}(1, v)=\mathbf{T}_{\mathrm{L} 1}{ }^{\mathrm{u}}(0, v)$ along the interval $0 \leq v \leq 1$. These mean that, the first, at $\mathbf{T}_{\mathrm{L} 1}(1, v)$ and $\mathbf{T}_{\mathrm{L} 2}(0, v)$ it must be

$$
\begin{align*}
{[\mathbf{c}+\lambda . \mathbf{l}]_{\mathrm{L} 1} } & =[\mathbf{c}]_{\mathrm{L} 2}  \tag{8a}\\
\Upsilon_{\mathrm{L} 1}(1, v) & =\Upsilon_{\mathrm{L} 2}(0, v) \tag{8b}
\end{align*}
$$

in plane $\left[\mathbf{v}_{1}, \mathbf{v}_{2}\right.$ ]. The second, along the interval $0 \leq v \leq 1$, the tangent vectors $\mathbf{L}_{1}{ }^{u}(u)$ and $\mathbf{L}_{2}{ }^{u}(u)$ of its center curves and the first derivation of the tube radius $\Upsilon_{\mathrm{L} 1}{ }^{\mathrm{u}}(u, v)$ and $\Upsilon_{\mathrm{L} 2}{ }^{\mathrm{u}}(u, v)$ have to

$$
\begin{align*}
\mathbf{L}_{1}{ }^{\mathrm{u}}(1) & =\sigma \mathbf{L}_{2}{ }^{\mathrm{u}}(0) \text { or } \quad \mathbf{l}_{1}=\sigma \mathbf{l}_{2}  \tag{9a}\\
\Upsilon_{\mathrm{L} 1}^{\mathrm{u}}(1, v) & =\Upsilon_{\mathrm{L} 2}{ }^{\mathrm{u}}(0, v) \tag{9b}
\end{align*}
$$

with $\sigma$ positive real scalar. When the center curve of pipe $\mathbf{T}_{2}(u, v)$ is a cubic Bézier curve of equation (6), then $\mathbf{B}^{\prime}(u)=3\left[\left(\mathbf{P}_{1}-\mathbf{P}_{o}\right)(1-u)^{2}+2\left(\mathbf{P}_{2}-\mathbf{P}_{1}\right)(1-u) \cdot u+\left(\mathbf{P}_{3}-\mathbf{P}_{2}\right) \cdot u^{2}\right], \mathbf{B}^{\prime}(0)=3\left[\left(\mathbf{P}_{1}-\mathbf{P}_{o}\right)\right]$ and the tangential continuity condition must be

$$
\begin{align*}
& {[\mathbf{c}+\lambda . \mathbf{I}]_{\mathrm{L} 1}=\left[\mathbf{P}_{\mathrm{o}}\right]_{\mathrm{L} 2}}  \tag{10a}\\
& \Upsilon_{\mathrm{L} 1}(1, v)=\Upsilon_{\mathrm{L} 2}(0, v)  \tag{10b}\\
& \mathbf{l}_{1}=3 \tau\left(\mathbf{P}_{1}-\mathbf{P}_{o}\right)  \tag{10c}\\
& \Upsilon_{\mathrm{L} 1}{ }^{\mathrm{u}}(1, v)=\Upsilon_{\mathrm{L} 2}{ }^{\mathrm{u}}(0, v) \tag{10d}
\end{align*}
$$

with $\tau$ positive real scalar.
Two tubular surface pieces $\mathbf{T}_{1}(u, v)$ and $\mathbf{T}_{2}(u, v)$ of the formulae (2) with its center curves $\boldsymbol{\Gamma}_{1}(u)$ and $\Gamma_{2}(u)$ of equation (6) or (7) in the same direction can be joined in the tangential continuity, if along the interval $0 \leq v \leq 1$, they verify the conditions

$$
\begin{align*}
& \mathbf{T}_{1}(1, v)=\mathbf{T}_{2}(0, v)  \tag{11a}\\
& \mathbf{t}_{1}(1)=\alpha \cdot \mathbf{t}_{2}(0)  \tag{11b}\\
& r_{1}^{\mathrm{u}}(1, v)=\Upsilon_{2}^{\mathrm{u}}(0, v) \tag{11c}
\end{align*}
$$

with $\alpha$ positive real scalar.
To Justify the implementation of equations (8-11), we show in the following connection of pipes. Let two parametric tubular surface pieces $\mathbf{T}_{1}(u, v)$ with its center curve $\mathbf{L}(u)=<-15,0,0>+$ 20.u. $\langle 1,0,0\rangle$ and $\mathbf{T}_{2}(u, v)$ with its center curve of equation (6) where $\left[\mathbf{P}_{o}=\langle 5,0,0\rangle, \mathbf{P}_{1}=\right.$ $\langle 15,0,0\rangle, \mathbf{P}_{2}=\langle 25,3,16\rangle, \mathbf{P}_{3}=\langle 35,14,10\rangle$ ]. Meanwhile $\Upsilon_{1}(u, v)=\Upsilon_{2}(u, v)=2+\cos (2 \pi u)-$ $\sin (2 \pi u)$. It is clear that $\mathbf{T}_{1}(u, v)$ and $\mathbf{T}_{2}(u, v)$ satisfy all conditions of equations (10) and the result is shown in Figure 2a. Other tangential connections of pipes can be showed in Figures 2b, 2c and 2d. Base on this calculation method, in the next section we discuss about the Frenet connection between two tubular pieces Bézier and Hermit of the equation (6) and (7) respectively.

The tangential continuity conditions in equation (11) of two pipe parts $\mathbf{T}_{1}(u, v)$ and $\mathbf{T}_{2}(u, v)$ are also in the moving trihedron continuity, if the triplet $\left[\mathbf{t}_{1}(1), \mathbf{n}_{1}(1), \mathbf{b}_{1}(1)\right]$ and $\left[\mathbf{t}_{2}(0), \mathbf{n}_{2}(0), \mathbf{b}_{2}(0)\right]$ of the center curves $\Gamma_{1}(u)$ and $\Gamma_{2}(u)$ coincide each other respectively and the second derivation of the direction $u$ of radius $\Upsilon_{1}(u, v)$ and $\Upsilon_{2}(u, v)$ at $\mathbf{T}_{1}(1, v)$ and $\mathbf{T}_{2}(0, v)$ are equal respectively. So, it must be

$$
\begin{align*}
& \mathbf{n}_{1}(1)=\beta \cdot \mathbf{n}_{2}(0)  \tag{12a}\\
& \mathbf{b}_{1}(1)=\gamma \cdot \mathbf{b}_{2}(0)  \tag{12b}\\
& \gamma_{1}^{\mathrm{uu}}(1, \mathrm{v})=Y_{2}^{\mathrm{uu}}(0, \mathrm{v}) \tag{12c}
\end{align*}
$$

with $\beta$ and $\gamma$ positive real scalars. The calculation of the unit vectors $\mathbf{t}, \mathbf{n}$ and $\mathbf{b}$ is as follows.
$\mathbf{B}^{\prime}(u)=\left\langle R_{x}(u), R_{y}(u), R_{z}(u)\right\rangle$
with

$$
\begin{aligned}
& R_{x}(u)=3\left[\left(P_{1 x}-P_{o x}\right) \cdot(1-u)^{2}+2\left(P_{2 x}-P_{1 x}\right)(1-u) \cdot u+\left(P_{3 x}-P_{2 x}\right) \cdot u^{2}\right] \\
& R_{y}(u)=3\left[\left(P_{1 y}-P_{o y}\right) \cdot(1-u)^{2}+2\left(P_{2 y}-P_{1 y}\right)(1-u) \cdot u+\left(P_{3 y}-P_{2 y}\right) \cdot u^{2}\right] \\
& R_{z}(u)=3\left[\left(P_{1 z}-P_{o z}\right) \cdot(1-u)^{2}+2\left(P_{2 z}-P_{1 z}\right)(1-u) \cdot u+\left(P_{3 z}-P_{2 z}\right) \cdot u^{2}\right] .
\end{aligned}
$$



Figure 2. Connection 2 pipes with its centers of line, Bézier and Hermit curves
$\mathbf{B}^{\prime \prime}(u)=\left\langle W_{x}(u), W_{y}(u), W_{z}(u)\right\rangle$
with

$$
\begin{aligned}
& W_{x}(u)=6\left[\left(P_{2 x}-2 P_{1 x}+P_{o x}\right) \cdot(1-u)+\left(P_{3 x}-2 P_{2 x}+P_{1 x}\right) \cdot u\right] \\
& W_{y}(u)=6\left[\left(P_{2 y}-2 P_{1 y}+P_{o y}\right) \cdot(1-u)+\left(P_{3 y}-2 P_{2 y}+P_{1 y}\right) \cdot u\right] \\
& W_{z}(u)=6\left[\left(P_{2 z}-2 P_{1 z}+P_{o z}\right) \cdot(1-u)+\left(P_{3 z}-2 P_{2 z}+P_{1 z}\right) \cdot u\right] .
\end{aligned}
$$

$\mathbf{H}^{\prime}(u)=\left\langle N_{x}(u), N_{y}(u), N_{z}(u)\right\rangle$
with

$$
\begin{aligned}
& N_{x}(u)=\mathrm{H}_{o x}\left(6 u^{2}-6 u\right)+\mathrm{H}_{1 x}\left(-6 u^{2}+6 u\right)+\mathrm{H}_{o x}^{u}\left(3 u^{2}-4 u+1\right)+\mathrm{H}_{1 x}^{u}\left(3 u^{2}-2 u\right) \\
& N_{y}(u)=H_{o y}\left(6 u^{2}-6 u\right)+H_{1 y}\left(-6 u^{2}+6 u\right)+H_{o y}^{u}\left(3 u^{2}-4 u+1\right)+H_{1 y}^{u}\left(3 u^{2}-2 u\right) \\
& N_{z}(u)=H_{o z}\left(6 u^{2}-6 u\right)+H_{1 z}\left(-6 u^{2}+6 u\right)+H_{o z}^{u}\left(3 u^{2}-4 u+1\right)+H_{1 z}^{u}\left(3 u^{2}-2 u\right) .
\end{aligned}
$$

$\mathbf{H}^{\prime \prime}(u)=\left\langle Z_{x}(u), Z_{y}(u), Z_{z}(u)\right\rangle$
with

$$
\begin{aligned}
& Z_{x}(u)=H_{o x}(12 u-6)+H_{1 x}(-12 u+6)+H_{o x}^{u}(6 u-4)+H_{1 x}^{u}(6 u-2) \\
& Z_{y}(u)=H_{o y}(12 u-6)+H_{1 y}(-12 u+6)+H_{o y}^{u}(6 u-4)+H_{1 y}^{u}(6 u-2) \\
& Z_{z}(u)=H_{o z}(12 u-6)+H_{1 z}(-12 u+6)+H_{o z}^{u}(6 u-4)+H_{1 z}^{u}(6 u-2) .
\end{aligned}
$$

```
\(\mathbf{k}_{B}=\left\langle M_{x}(u), M_{y}(u), M_{z}(u)\right\rangle\)
with
            \(M_{x}=\left[s^{2} \cdot W_{x}(u)-\left[R_{x}^{2}(u) \cdot W_{x}(u)+R_{x}(u) \cdot R_{y}(u) \cdot W_{y}(u)+R_{x}(u) \cdot R_{z}(u) \cdot W_{z}(u)\right]\right] / s^{4}\)
            \(M_{y}=\left[s^{2} \cdot W_{y}(u)-\left[R_{y}(u) \cdot R_{x} \cdot W_{x}(u)+R_{y}^{2}(u) \cdot W_{y}(u)+R_{y}(u) \cdot R_{z}(u) \cdot W_{z}(u)\right]\right] / s^{4}\)
            \(M_{z}=\left[s^{2} \cdot W_{z}(u)-\left[R_{z}(u) \cdot R_{x}(u) \cdot W_{x}(u)+R_{z}(u) \cdot R_{y}(u) \cdot W_{y}(u)+R_{z}{ }^{2}(u) \cdot W_{z}(u)\right]\right] / s^{4}\)
            \(s=\left[R_{x}{ }^{2}(u)+R_{y}^{2}(u)+R_{z}^{2}(u)\right]^{1 / 2}\).
```

$\mathbf{k}_{H}=\left\langle S_{x}(u) ; S_{y}(u) ; S_{z}(u)\right\rangle$
with

$$
\begin{aligned}
& S_{x}=\left[n^{2} \cdot Z_{x}(u)-\left[N_{x}^{2}(u) \cdot Z_{x}(u)+N_{x}(u) \cdot N_{y}(u) \cdot Z_{y}(u)+N_{x}(u) \cdot N_{z}(u) \cdot Z_{z}(u)\right]\right] / n^{4} \\
& S_{y}=\left[n^{2} \cdot Z_{y}(u)-\left[N_{y}(u) \cdot N_{x} \cdot Z_{x}(u)+N_{y}^{2}(u) \cdot Z_{y}(u)+N_{y}(u) \cdot N_{z}(u) \cdot Z_{z}(u)\right]\right] / n^{4} \\
& S_{z}=\left[n^{2} \cdot Z_{z}(u)-\left[N_{z}(u) \cdot N_{x}(u) \cdot Z_{x}(u)+N_{z}(u) \cdot N_{y}(u) \cdot Z_{y}(u)+N_{z}^{2}(u) \cdot Z_{z}(u)\right]\right] / n^{4} \\
& n=\left[N_{x}^{2}(u)+N_{y}^{2}(u)+N_{z}^{2}(u)\right]^{1 / 2} .
\end{aligned}
$$

So, the unit vector tangent, normal and binormal of the Bezier center curve is

$$
\begin{align*}
\mathbf{t}_{B}= & (1 / \mathrm{s})\left\langle R_{x}(u), R_{y}(u), R_{z}(u)\right\rangle  \tag{13a}\\
\mathbf{n}_{B}=\langle & \left\langle\left(M_{x}(u) / s_{o}\right),\left(M_{y}(u) / s_{o}\right),\left(M_{z}(u) / s_{o}\right)\right\rangle  \tag{13b}\\
\mathbf{b}_{B}=\langle & \left\langle R_{y}(u) \cdot M_{z}(u)-M_{y}(u) \cdot R_{z}(u)\right] /\left(s \cdot s_{o}\right), \\
& {\left[M_{x}(u) \cdot R_{z}(u)-R_{x}(u) \cdot M_{z}(u)\right] /\left(s . s_{o}\right), } \\
& {\left.\left[R_{x}(u): M_{y}(u)-M_{x}(u): R_{y}(u)\right] /\left(s \cdot s_{o}\right)\right\rangle } \tag{13c}
\end{align*}
$$

with $s_{o}=\left[M_{x}{ }^{2}(u)+M_{y}{ }^{2}(u)+M_{z}{ }^{2}(u)\right]^{1 / 2}$. On the other hand, we have the unit vector tangent, normal and binormal of the Hermit center curve

$$
\begin{align*}
& \mathbf{t}_{H}=1 / n<N_{x}(u) ; N_{y}(u) ; N_{z}(u)>  \tag{14a}\\
& \mathbf{n}_{H}=<\left.S_{x}(u) / n_{o}, S_{y}(u) / n_{o}, S_{z}(u) / n_{o}\right\rangle  \tag{14b}\\
& \mathbf{b}_{H}=<\left[N_{y}(u) \cdot S_{z}(u)-S_{y}(u) \cdot N_{z}(u)\right] /\left(n . n_{o}\right), \\
& {\left[S_{x}(u) \cdot N_{z}(u)-N_{x}(u) \cdot S_{z}(u)\right] /\left(n . n_{o}\right), } \\
& {\left.\left[N_{x}(u) \cdot S_{y}(u)-S_{x}(u) \cdot N_{y}(u)\right] /\left(n . n_{o}\right)\right\rangle } \tag{14c}
\end{align*}
$$

with $n_{o}=\left[S_{x}^{2}(u)+S_{y}^{2}(u)+S_{z}^{2}(u)\right]^{1 / 2}$.

## 4. Conclusions

We have formulated the continuous connection of various pipe shapes which its center curves $\Gamma(u)$ are defined by line, Bézier and Hermit curves. We can conclude that two parametric tubular surface pieces $\mathbf{T}_{1}(u, v)$ and $\mathbf{T}_{2}(u, v)$ connect continuously order-1, if they verify the conditions: first, $\mathbf{T}_{1}(1, v)=\mathbf{T}_{2}(0, v)$. Second, its tangent vectors of center curve $\boldsymbol{\Gamma}_{1}(u)$ and $\boldsymbol{\Gamma}_{2}(u)$ at the values $\mathbf{T}_{1}(1, v)$ and $\mathbf{T}_{2}(0, v)$ and its tangent vectors of longitudinal boundary curve along the cross section boundary curve are equal. They are also in the moving trihedron continuity, if its triplet $[\mathbf{t}, \mathbf{n}, \mathbf{b}]$ of the center curves at $\Gamma_{1}(1)$ and $\Gamma_{2}(0)$ coincide each other respectively and the second derivation of the direction $u$ of radius $\Upsilon_{1}(u, v)$ and $\Upsilon_{2}(u, v)$ respectively at $\mathbf{T}_{1}(1, v)$ and $\mathbf{T}_{2}(0, v)$ are equal.

The continuous connection of various pipe shapes have been introduced. The interesting thing to discuss ahead is how to build a complete pipe using some parts of pipes in various thickness. Also, the connection have to be varied and continuous.

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