Diregularity of digraphs of out-degree three and order two less than Moore bound

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Abstract. It is easy to show that any digraph with out-degree at most $d \geq 2$, diameter $k \geq 2$ and order $n = d + d^2 + \ldots + d^k - 1$, that is, two less than Moore bound must have all vertices of out-degree $d$. In other words, the out-degree of the digraph is constant ($= d$). However, establishing the diregularity or otherwise of the in-degree of such a digraph is not easy. It was proved that every digraph of out-degree at most two, diameter $k \geq 3$ and order two less than the Moore bound is diregular [8]. In this paper, we consider the diregularity of digraphs of out-degree at most three, diameter $k \geq 3$ and order two less than the Moore bound.

1 Introduction and preliminaries

A directed graph or digraph $G$ is a pair of sets $(V, A)$ where $V$ is a finite nonempty set of distinct elements called vertices, and $A$ is a set of ordered pairs $(u, v)$ of distinct vertices $u, v \in V$ called arcs.

The order $n$ of a digraph $G$ is the number of vertices in $G$, that is, $n = |V|$. An in-neighbour (respectively out-neighbour) of a vertex $v$ in $G$ is a vertex $u$ (respectively $w$) such that $(u, v) \in A$ (respectively $(v, w) \in A$). The set of all in-neighbours (respectively out-neighbours) of a vertex $v$ is denoted by $N^{-}(v)$ (respectively $N^{+}(v)$). The in-degree (respectively out-degree) of a vertex $v$ is the number of its in-neighbours (respectively out-neighbours). We denote by $d^{-}(v)$ the in-degree of $v$ in $G$. If the in-degree equals the out-degree ($=d$, say) for every vertex in $G$, then $G$ is called a diregular digraph of degree $d$.

A $v_0 - v_l$ walk of length $l$ in a digraph $G$ is an alternating sequence $v_0a_1v_1a_2\ldots a_lv_l$ of vertices and arcs in $G$ such that $a_i = (v_{i-1}, v_i)$ for each $i$, $1 \leq i \leq l$. A walk is closed if $v_0 = v_l$. If all the vertices of a $v_0 - v_l$ walk are distinct, then such a walk is called a path. A cycle is a closed walk with all vertices and edges are distinct (except the first and the last vertices).

The distance from vertex $u$ to $v$, denoted by $\delta(u, v)$, is the length of the shortest path from vertex $u$ to vertex $v$. Note that in general $\delta(u, v)$ is not necessarily