On two conjectures concerning vertex-magic total labelings of generalized Petersen graphs

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Abstract

A vertex-magic total labeling of a graph with \( v \) vertices and \( e \) edges is defined as a one-to-one map taking the vertices and edges onto the integers \( 1, 2, \ldots, v+e \) with the property that the sum of the labels on a vertex and the labels on its incident edges is a constant, independent of the choice of vertex.

In this paper we give a vertex-magic total labeling for the prism \( D_n \) for all \( n \geq 3 \); and a vertex-magic total labeling for the generalized Petersen graphs \( P(n, m) \) for \( n \geq 3 \), \( 1 \leq m \leq \lfloor \frac{2n-1}{3} \rfloor \), and \( n \) and \( m \) coprime.

1 Introduction

A vertex-magic total labeling of a graph \( G = (V, E) \) with \( v \) vertices and \( e \) edges is an assignment of the integers from 1 to \( v+e \) to the vertices and edges of \( G \) with the property that the sum of the label on a vertex and the labels on its incident edges is a constant, independent of the choice of vertex. More formally, a one-to-one map \( \lambda \) from \( E \cup V \) onto the integers \( \{1, 2, \ldots, e+v\} \) is a vertex-magic total labeling if there is a constant \( k \) so that for every vertex \( x \),

\[
\lambda(x) + \sum \lambda(xy) = k
\]

(1)

where the sum is over all vertices \( y \) adjacent to \( x \). The constant \( k \) is called the magic constant for \( \lambda \). This notion was introduced in [5]. We note that not every graph has a vertex-magic total labeling. For the graph \( K_2 \), since \( \lambda(x) \neq \lambda(y) \), then \( \lambda(x) + \lambda(xy) \neq \lambda(y) + \lambda(xy) \), and so no vertex-magic total labeling is possible. Another example is \( K_{m,n} \), for \( n > m + 1 \), see [5].