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We are honored to bring you this collection of articles from the 3rd International Conference on Mathematical Sciences and Statistics (ICMSS2018) that was held in Putrajaya, Malaysia from 6th to 8th February 2018.

The primary focus of this conference was to bring together academicians, researchers and scientists for knowledge sharing in various areas of Mathematics and Statistics. The ICMSS2018 served as a good platform for the scientific community where almost 200 participants met to exchange ideas.

During the three days of conference, the researchers presented the most recent discoveries in Mathematics and Statistics as well as established networking for possible joint researches and collaborations among the participants.

The editors would like to thank the participants who have contributed to the volume, which is a selected collection of 84 papers. We also express our gratitude to every staff of the Department of Mathematics for their unwavering commitment as the conference organizer and the management of Faculty of Science, Universiti Putra Malaysia for their unfailing support towards ICMSS2018.

Lastly, we are most indebted for the generous support given by Institute for Mathematical Research (INSPEM), Malaysian Mathematical Sciences Society (PERSAMA), Malaysian Institute of Statistics (ISM), OEMS Intipakar Corporation Sdn. Bhd. and Bizit Systems (M) Sdn. Bhd.

20 September 2018

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Table of contents

Volume 1132

2018

[◀ Previous issue](#) [Next issue ▶](#)

**3rd International Conference on Mathematical Sciences and Statistics 6–8
February 2018, Le Meridien Putrajaya, Malaysia**

Accepted papers received: 07 November 2018

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Preface

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Peer review statement

[+ Open abstract](#) [View article](#) [PDF](#)

Papers

OPEN ACCESS 012001

Bounds on the lengths of certain series expansions

Yanapat Tongron, Narakorn Rompurk Kanasri and Vichian Laohakosol

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012002

The parameterization of nice and Q-nice polynomials with four roots

H Anton, S H Sapar and M A M Johari

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012003

Parikh matrices of arrays under Dejean array morphism

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012004

A stability result about a functional equation of Drygas on an Abelian group

Janyarak TongsoPorn and Vichian Laohakosol

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012005

Cyclic normal fuzzy neutrosopic soft G -modular structures acting on a group

P Jayaraman

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012006

Two-dimensional picture arrays and Parikh q -matrices

Somnath Bera, Kalpana Mahalingam, Linqiang Pan and K. G. Subramanian

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012007

Two Properties of Pseudo-Polynomials over a Galois Field

Rattiya Meesa, Vichian Laohakosol, Tuangrat Chaichana and Boonrod Yuttanan

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012008

Numerical Integration Based on Linear Legendre Multi Wavelets

Mohammad Hasan Abdul Sathar, Ahmad Fadly Nurullah Rasedee, Anvarjon A. Ahmedov and Muhammad Asyraf Asbullah

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012009

Variational inequality problems for total quasi-asymptotically nonexpansive mapping in Hilbert spaces

L B Mohammed and A Kiliçman

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012010

On a class of Entire function represented by the Dirichlet series in two variables

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012011

A new study of generalized Ma-Minda type class of meromorphic functions

F Ghanim

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012012

Parikh matrices and M -ambiguity sequence

Ghajendran Poovanandran and Wen Chean Teh

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012013

On the summability of the spectral expansions associated with the elliptic differential operators

A Rakhimov

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012014

An embedded 4(3) pair explicit two derivative Runge-Kutta-Nyström method for solving $y''(x) = f(x, y, y')$

T S Mohamed, N Senu, N M A Nik Long and Z B Ibrahim

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012015

Soliton solutions for Fisher equation by using simplified Hirota's method

Feras Shatat

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012016

Exponentially-fitted forth-order explicit modified Runge-Kutta type method for solving third-order ODEs

N Ghawadri, N Senu, F Ismail and Z B Ibrahim

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS 012017

Numerical solution for stiff initial value problems using 2-point block multistep method

N Mohamad Noor, Z B Ibrahim and F Ismail

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012018

Exponentially-fitted Fourth-Order Taylor's Algorithm for Solving First-Order ODEs

M A Akanbi and A S Wusu

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012019

Modified HPM for high-order linear fractional integro- differential equations of Fredholm-Volterra type

Z K Eshkuvatov, M H Khadijah and B M Taib

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012020

Performance of space-time coupled least-squares spectral element methods for parabolic problems

P Biswas, N Kishore Kumar and Anil Kumar Kar

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012021

New travelling solitary wave solutions for an evolution equation by three schemes

A J Mohamad Jawad and M J Abu-Al Shaeer

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012022

An epidemic model of tuberculosis with vaccine control in Yogyakarta region Indonesia

D Lestari, A Dhoruri and E R Sari

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012023

Biological Experiments Based on Fractional Integral Equations

Faten H Damag and Adem Kilicman

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012024

Solutions to neutral partial functional differential equations with functional delay

Md. Maqbul

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012025

The product of virtually nonexpansive maps and their fixed points

K Pomdee, G Sunyeekhan and P Hirunmasuwan

[+ Open abstract](#) [View article](#) [PDF](#)

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012026

Heat transfer and axisymmetric stagnation point flow due to a shrinking vertical plate in a nanofluid with slip effects

M A Kardri, N Bachok, N M Arifin and F M Ali

[+ Open abstract](#) [View article](#) [PDF](#)

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012027

Thermal Non-equilibrium Double Diffusive Convection in a Maxwell Fluid with Internal Heat Source

A.A. Altawallbeh, I. Hashim and A.A. Tawalbeh

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012028

Riemannian Geometry of Ising Model in the Bethe Approximation

Riza Erdem

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012029

Unsteady stagnation-point flow and heat transfer over an exponential stretching sheet in Copper-water nanofluid with slip velocity effect

N F Dzulkipli, N Bachok, I Pop, N A Yacob, N M Arifin and H Rosali

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012030

Half Circle Position for Arc Cracks in Half Plane

N. R. F. Elfakhakhre, N. M. A. Nik long and Z. K. Eshkuvatov

[+ Open abstract](#) [View article](#) [PDF](#)

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012031

The enhancement of high school students' mathematical reasoning

through team-assisted individualization

S F Tauran

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012032

Epistemological obstacles in mathematical abstraction on abstract algebra

Toto Subroto and Didi Suryadi

[+ Open abstract](#) [View article](#) [PDF](#)

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012033

Student's conceptions and geometry problem-solving of the distance in cube

E D Minarti, Wahyudin and F Alghadari

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012034

Assessing misconception reasoning and communication statistical about variability among Madrasah Tsanawiyah students

Iyam Maryati and Nanang Priatna

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012035

GeoEnzo utilization as mathematics learning media with contextual approach to increase geometry understanding

R Mauladaniyati and D Kurniawan

[+ Open abstract](#) [View article](#) [PDF](#)

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012036

Analysis on geometry skills of junior high school students on the concept congruence based on Van Hiele's geometric thinking level

Reni Astuti, Didi Suryadi and Turmudi

[+ Open abstract](#) [View article](#) [PDF](#)

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012037

Analysis of mathematical abstraction on concept of a three dimensional figure with curved surfaces of junior high school students

N Fitriani, D Suryadi and D Darhim

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012038

Effects of gender and school type on attitudes towards mathematics

Alyaa Naadiah Mohamed and Fatimah Abdul Razak

[+ Open abstract](#) [View article](#) [PDF](#)

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012039

Experiential Statistics Learning with RStudio: Study on Students' Engagement

N R Salim, K Gopal and A F M Ayub

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012040

Developing student character of preservice mathematics teachers through blended learning

Dahlia Fisher and Yaya Sukjaya Kusumah

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012041

The effectiveness of metacognitive learning in enhancing student's mathematical analysis

M Hutajulu and W Wahyudin

[+ Open abstract](#) [View article](#) [PDF](#)

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012042

Influence of self-efficacy and attitudes towards statistics on undergraduates' statistics engagement in a Malaysian public university

K Gopal, N R Salim and A F M Ayub

[+ Open abstract](#) [View article](#) [PDF](#)

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012043

Identify student mathematical understanding ability through direct learning model

Tina Sri Sumartini and Nanang Priatna

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012044

The analogical reasoning analysis of Pesantren students in geometry

Sendi Ramdhani and Didi Suryadi

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012045

Analysis of mathematical modelling ability of line equations of junior high school students

A Yuliani and Y S Kusumah

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012046

The obstacles of geometric problem-solving on solid with vector and triangle approach

F Alghadari and T Herman

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012047

Analysis of internal and external mathematical representation ability to senior high school students in Indonesia

Elsa Komala and Didi Suryadi

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012048

Mathematical mindsets: the abstraction in mathematical problem solving

Luki Luqmanul Hakim and Elah Nurlaelah

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012049

Mathematical abstraction ability of prospective math teacher students

J D Putra, D Suryadi and D Juandi

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012050

The correlation between working memory and students' mathematical difficulties

Iyan Rosita Dewi Nur, Tatang Herman, Tina Hayati Dahlan and Uba Umbara

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012051

The students' mathematical concept understanding ability through cooperative learning type jigsaw assisted visual media

S Sumarni, D Darhim, S Fatimah, N Priatna, A Anjelita and A Taufik

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012052

Analysing categories of mathematical proficiency based on Kilpatrick opinion in junior high school

Vara Nina Yulian and Wahyudin

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012053

Deterministic modeling of the transmission dynamics of intramammary infections

Amira Rachah, Gunnar Dalen, Håvard Nørstebø, Olav Reksen and John W. Barlow

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012054

Pursuit game problem of an infinite system of differential equations with geometric and integral constraints

Usman Waziri, Gafurjan Ibragimov, Idham Arif Alias and Zarina Bibi Ibrahim

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012055

New search direction of steepest descent method for large-scaled unconstrained optimization problem

Siti Farhana Husin, Mustafa Mamat, Mohd Asrul Hery Ibrahim and Mohd Rivaie

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012056

Newton method with explicit group iteration for solving large scale unconstrained optimization problems

K Ghazali, J Sulaiman, Y Dasril and D Gabda

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012057

Stationary queue length distribution of a continuous-time queueing system with negative arrival

C H Chin, S K Koh, Y F Tan, A H Pooi and Y K Goh

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012058

A mathematical model with isolation for the dynamics of Ebola virus

Amira Rachah

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012059

The Shapley weighting vector-based neutrosophic aggregation operator in DEMATEL method

A Awang, A T Ab Ghani, L Abdullah and M F Ahmad

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012060

An approximate method for solving fractional partial differential equation by using an embedding process

E Ziaei, M H Farahi, A Ahmadian, N Senu and S Salahshour

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012061

Differential inequalities related to Salagean type integral operator involving extended generalized Mittag-Leffler function

Hiba Fawzi Al-Janaby and Muhammad Zaini Ahmad

[+](#) [Open abstract](#) [View article](#) [PDF](#)

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012062

Forcing geodesic number of a fuzzy graph

S Rehmani and M S Sunitha

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012063

Block Diagonal Preconditioners for an Image De-blurring Problem with Fractional Total Variation

Adel Al-Mahdi and Faisal Fairag

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012064

A note on simulation of reaction systems by the minimal ones

Wen Chean Teh

[+](#) [Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012065

Classes of harmonic univalent functions convex in one direction

Rana Al-Khal and Khalifa Al-Shaqsi

[+ Open abstract](#) [View article](#) [PDF](#)

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012066

Enhanced Markov-based model for the availability analysis of distributed software and hardware systems

Lawan Abdulwahab, Jabir Tukur Abdullahi and Ibrahim Yusuf

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012067

Model-building on survivability of upper gastrointestinal bleed patient's

Khuneswari Gopal Pillay and Siti Aisyah Mohd Padzil

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012068

Purchasing power parity for the MAVINS

Niri Martha Choji and Siok Kun Sek

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012069

Analysis of international visitor arrivals in Bali: modeling and forecasting with seasonality and intervention

Joy Melchisedec Pierre Mangindaan and Tupaluck Kriyakerne

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012070

E-Statistics as an instrument for reliable source of data

A A Gwani and A Aliyu

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012071

Estimating a finite population mean under random non response in two stage cluster sampling with replacement

Nelson Kiprono Bii, Christopher Ouma Onyango and John Odhiambo

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012072

Bootstrapping technique in structural equation modeling: a Monte Carlo study

Nor Iza Anuar Razak, Zamira Hasanah Zamzuri and Nur Riza Mohd Suradi

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012073

The Bregman-divergence universal portfolio associated with a convex polynomial

Choon Peng Tan and Yap Jia Lee

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012074

A fractional difference returns for stylized fact studies

Rosmanjawati Abdul Rahman and Jibrin Sanusi Alhaji

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012075

Prediction of hydrocarbon depth for seabed logging (SBL) application using Gaussian process

Muhammad Naeim Mohd Aris, Hanita Daud and Sarat Chandra Dass

[+ Open abstract](#) [View article](#) [PDF](#)

OPEN ACCESS

012076

Generalized dynamic principal component for monthly nonstationary stock market price in technology sector

Yusrina Andu, Muhammad Hisyam Lee and Zakariya Yahya Algamal

[+ Open abstract](#) [View article](#) [PDF](#)

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012077

On the development of the GE and the GGE interaction Biplot in the RCIM model and the evaluation of its' robustness to the outlying observations

Alfian Futuhul Hadi, H Sadiyah and M Hasan

[+ Open abstract](#) [View article](#) [PDF](#)

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012078

Analysis and Assessment of Boxplot Characters for Extreme Data

Babangida Ibrahim Babura, Mohd Bakri Adam, Abdul Rahim Abdul Samad, Anwar Fitrianto and Bashir Yusif

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012079

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012080

The study of properties on generalized Beta distribution

D W W Ng, S K Koh, S Z Sim and M C Lee

[+ Open abstract](#) [View article](#) [PDF](#)

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012081

Perceived acceptability towards self-sampling for Human Papillomavirus (HPV) using Rasch measurement model: study revisited

Anis Syakira Jailani, Zamalia Mahmud and Nik Nairan Abdullah

[+ Open abstract](#) [View article](#) [PDF](#)

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012082

Information content of option-implied probabilities

Greg Orosi

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012083

Explicit Formula for Conditional Expectations of Product of Polynomial and Exponential Function of Affine Transform of Extended Cox-Ingersoll-Ross Process

Phiraphat Sutthimat, Khamron Mekchay and Sanae Rujivan

[+ Open abstract](#) [View article](#) [PDF](#)

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012084

A study on the variable sampling interval EWMA \bar{X} chart when the process parameters are unknown

L V Ong, W L Teoh, M B C Khoo, Z L Chong and W C Yeong

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On the development of the GE and the GGE interaction Biplot in the RCIM model and the evaluation of its' robustness to the outlying observations

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Abstract. Our recent statistical research on modeling of the two-ways table data was focused especially on the robustness of Row Column Interaction Model (RCIM). It has been showed that the RCIM model provide better result in fitting the data with outliers better than others even for Normal distribution. In this paper we focus on the influenced of the outlying observations to the visualization of the interaction effects in the RCIM modeling. We proposed the Genotype by Environment (GE) and the Genotype and Genotype by Environment (GGE) graphics display by a Biplot on the RCIM model. We also evaluate the influence of the outlying observations to the two kinds of Biplot of GEI by adding the outlying observations to the data. According to the Mean Square Error (MSE) and the Procrustes analysis, the GGE Biplot of RCIM has better result than the GE. The GE Biplot fails to accommodate inflation variance by the presence of a single environmental outliers with large percentage. In addition, the GE Biplot has difficulties to hold total percentage of the variance explained.

1. Introduction

As part of the national strategy of food sufficiency, plant breeding provided some useful information determining the suitable genotype in the variety development. In wide archipelagic agricultural country area like Indonesia, not every region has a similar condition. There were some varieties of cultivar, that cannot grow well in any region. The variation of the environment may lead in observations having different characteristic to the other observations, known as outlier. Such outliers often excluded from the analytical data processing. in some cases of plant breeding research, the outliers have very useful information.

Various statistical methods have been developed to assess the Genotypes by Environments Interaction (GEI), the two of those are the Genotype by Environment (GE) and the Genotype and Genotype by Environment (GGE) Biplots. Biplot is an appropriate visualization tool for describing interactions in the data. The GE commonly used in the Additive Main Effects and Multiplicative Interactions (AMMI) model, it is considered an effective tool for determining the pattern of the GEI, graphically. While the GGE uses the environmentally centralized data, that is the $G + GE$, the GE



Biplot AMMI is known as with double-centered data. The AMMI analysis separates G from GE first and then puts them together again, whereas GGE biplot analysis deals with G+GE directly. Therefore, explicit separation of G from GE in AMMI analysis does not lead to the conclusion that it is superior to GGE biplot analysis [1].

The GE and the GGE Biplot were almost similar, they decompose the residual matrix by the Singular Value Decomposition (SVD) method then plotting the two main components into the two-dimensional superimposed graph. The difference is just that the GE Biplot has balanced weight (singular value) in each genotype and environmental score whereas the GGE gives an additional weight to the environmental score. But, this will result in different advantages in each Biplot. One unique merit of a GGE Biplot is that it can graphically show the which-won-where patterns of the data [2].

In more general overview of statistical modelling, there is an overlapping methodology of AMMI model, that is the RCIM of Yee and Hadi [3]. The RCIM model was identical to AMMI Model. The RCIM is also used for modelling count data that was potentially has unique robustness to outlier in Poisson data distribution [4]. The RCIM is an extension of the Reduced-Rank Vector Generalized Model (RR-VGLM) where the first linear predictor is modeled by the sum of row effect, column effect and the row by column interactions effect. The big issue here is that we knew that SVD is vulnerable to the outliers [5], so the construction of robust visualization of the interaction effects in the RCIM model is needed to be investigated.

This research was conducted to evaluate the robustness of RCIM model in the point of view of its graphical representative of the interactions by the GE and the GGE Biplot, especially in the case of there were outlying observations on the data. Our recent statistical research on the RCIM model's robustness, it was shown that the RCIM model provide a better result in fitting the data with outliers rather than others, even for normally distribution data. For further reading please see the detail in [4]. We proposed here the GE and the GGE Biplot on the RCIM Model, and then evaluate the influence of the outlying observations to the two kinds of Biplot. We now will focus on the visualization of the interaction and the effect of the outlying observations in it.

2. Methodology

This research used the data of Yan [1] with a simple scheme of simulation about outliers. This data was originally available on *GGEbiplotGUI* package, which has 18 variants of genotypes determined from the 9 different locations or environments. We then conducted a simple scheme of simulation for adding outliers to the data. The outliers were added to the data, placed randomly as we conducted before on [5], for 2%, 5% and 10% of outlier of the whole cells in the data matrix. In other words, we put the number of outliers 4, 7, and 17 respectively, from the total number of 162 observations. The outliers were generated randomly following the normal distribution [8]:

$$N(\mu_j + k\sigma_j, \sigma_j^2)$$

where

μ_j is the average value of the data for the j -th column.

σ_j^2 is the variance of the data.

k is a constant value of the magnitude of the outliers, $k= 1, 2, 3$.

2.1 Outlier

Outlier is defined as the part of observation which has different characteristics from most corresponding observation data set. An observation is considered as an outlier when its value of the k -multiplied of standard deviation is greater than its original mean, where the k is greater than 3. Mathematically, it can be expressed by the following [9]:

$$y_i \geq \mu_j + k \times stdev(y_i)$$

The value of k is the magnitude of the outlier, it shows how far the outlier value from its original mean. In this case, there are three values of k , i.e. $k=1, 2, 3$, that were used to evaluate the magnitude of the outlier. Furthermore, we simulated three scenarios of the number of outlier as proportion to the total cell of data matrix: there are 2%, 5% and 10%. Subsequently, the outlier then be placed on the data, follow [10] that proposed some placement methods, some of them which are our interest here:

1. *Scattered Outliers*

The (scattered) outlier will be allocated randomly in some representing positions, by choosing genotype (row) and environment (column) randomly. The outlier then was being placed on the certain row and column. The next outliers were placed at the same fashion.

2. *Single Environment Outliers*

Firstly, the outlier was placed randomly by choosing column (environment) randomly then the others following outliers were placed on the chosen row (genotype), which had been chosen randomly, until they filled in all elements of its column.

2.2 *RCIM for GE and GGE Biplot in R*

The GE and GGE Biplot were built from SVD of the residuals of the RCIM2 Model. However, there is a different model between the GE Biplot and the GGE Biplot of RCIM. We used the `rcim` command with `rank=0` --that is similar as the `vglm` command in VGAM packages-- to estimate the model [3]. The RCIM with normal distribution for the GE model will run the `rcim` with `gaussianff` family function:

```
rcim(data, family = gaussianff(), Rank = 0)
```

while the RCIM with normal distribution for the GGE model will use the `vglm` with `gaussianff` family function:

```
vglm(yield~G, family = gaussianff(), data)
```

after that we use decomposed the residual using the `svd` for both model respectively to get the first 2 principal components (PC1 and PC2) than we plotted it by the `ggplot2` packages.

2.3 *Procrustes*

Two ordinations could be similar. Two ordinations are said to be similar if they are only distinguished by a certain transformation we can find. Unfortunately, it is hard to find such a congruence, since the usually axes have slightly different orientation and scaling. The best way to compare ordinations is to use the procrustes rotation [11]. The procrustes aims to compare two configurations. In principle, to see the similarity of the shape and size of the two configurations, one of the configurations is fixed, while the other is transformed to fit the first one [12]. This process is interpretation by figure 1:

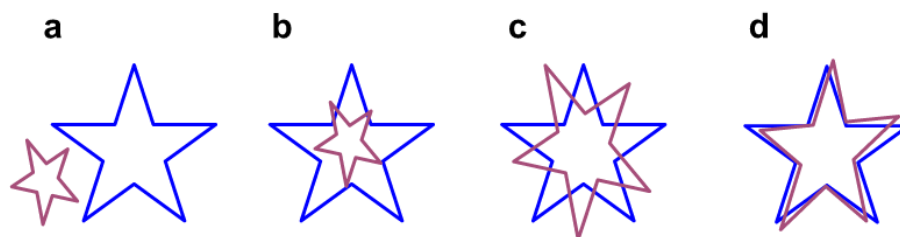


Figure 1. The procrustes superimposition of two shapes (a) begins by translating them to superimpose their centroids (b) before scaling (dialating) (c) and rotating them (d) to maximize their coincidence [12].

The procrustes rotates a configuration to maximum similarity with another configuration and also tests the non-randomness between two configurations. Procrustes rotation rotates a matrix to maximum similarity with a target matrix minimizing sum of squared differences. Procrustes rotation is typically used in comparison of ordination results. It is particularly useful in comparing alternative solutions in multidimensional scaling. The Procrustes function returns an object of class Procrustes with items. Function `protest` inherits from Procrustes then provides some items interesting: (1) The Sum of

Squared (SS) measure the differences between X and (rotated) Y. The smaller SS the more similar between the two matrices. (2) The “signif” is the p-value that the smaller p-value, the more similar the two matrices were [13].

3. Results

3.1 The influence of the outlier on the MSE

For this dataset without any outlier, Tables 1a & 1b show us that the GGE model was provided smaller MSE than the GE model. This means that by default, the GGE model was fitting the data better than the GE one. So does when we took a look to the result of simulated data with scattered or single environment outliers, we can see here that generally, the GGE model was fit the data properly. This indicate that potentially, the GGE model had robustness to the outlier better than the GE one.

The MSE was inflated to become higher than before in the original data without outlier. There was different influence corresponding to the GE or the GGE model; the outlier was seen to affect the inflation of the MSE of the GE model higher than that of the GGE model. Generally, either for the GE or the GGE model, there was a tendency that the higher value of outlier the higher MSE we got. Specifically, for the GE model, the more number of outliers in the data the larger value of MSE, either of scattered outliers or single environment outliers. We can say here that both the GE and the GGE model still facing the vulnerability of scattered outliers. But not so for the GGE model by the single environment outliers, there was something interesting here. For all k means, lower, middle or higher value of outliers, there was decreasing MSE with more percentage of 10% of single environment outliers. One may pay attention to the right below corner cell of table 1b at, bold printed. This indicated that the GGE model potentially hold the robustness to the SE outliers.

Table 1a. The MSE of the GE model.

Outlier	Percentage of outliers	The magnitude of the outlier		
		$k = 3$	$k = 10$	$k = 15$
Without Outlier	0%	0.830	0.830	0.830
Scattered Outliers	2%	1.130	2.279	3.103
	5%	1.083	3.292	6.838
	10%	1.281	5.780	12.367
Single Environment (SE) Outliers	2%	0.972	2.016	3.051
	5%	1.131	3.190	5.431
	10%	2.325	7.302	9.310

Table 1b. The MSE of the GGE model.

Outlier	Percentage of outliers	The magnitude of the outlier		
		$k = 3$	$k = 10$	$k = 15$
Without Outlier	0%	0.123	0.123	0.123
Scattered Outliers	2%	0.282	1.336	2.460
	5%	0.313	2.420	5.528
	10%	0.514	4.444	9.586
Single Environment (SE) Outliers	2%	0.214	0.990	2.301
	5%	0.235	1.215	3.042
	10%	0.180	0.470	0.826

3.2 The Procrustes: The influence of the outliers to the Biplot configuration

Procrustes is applied to test how far two configurations differ each other's. In this case, to test the influence of the outlier on the RCIM model, we verify the difference between the residual matrix of RCIM2 from the original data versus it's from the simulated data with outlier (scattered and single environment outliers). For small value scattered outlier, all of the Procrustes's p -value are slightly significant, it means that the small value outliers, both scattered and single environment outliers, was less influential on the RCIM Biplot either for GE or GGE model. It is indicating the potential robustness of RCIM Biplot to the small value of outlier. Table 2 show the similarity result as the data without outliers when the value of $k = 3$ with any percentage of scattered outliers, and so were the single environment outliers. But it was no longer parallel result with the higher value and percentage of outliers, the RCIM2 of the GGE model provide different result. But for higher value on scattered outliers $k=10$, the more number of outliers the more tendencies to get higher p -value in Procrustes analysis, that means the more influence impacted in the Biplot of the GGE model.

In the case of focusing on single environment outliers, Biplot of the GGE RCIM2 model with outliers also provides the similar result as RCIM2 model without outlier when the value of $k = 3$. But, there is some interesting information here, for higher value of single environment outlier, there was still possibility of robustness of the GGE Biplot of RCIM2 model. The highest value outlier $k=10$ and $k=15$ did not affect the Biplot of GGE when there were 10% outliers in the simulated data.

The procrustes sum of square between two residuals from GE model with outlier (scatter and single environment) and GE model without outlier show that the results are almost similar where the model with small value of outlier ($k = 3$) provides the similar result as the data without outliers. When we focus on 10% single environment outlier from GGE model and GE model, the residuals model with this single environment outlier and without outlier are almost similar. This indicates that RCIM is robustness with single environment outlier.

Table 2. The p -value of Procrustes of the GGE Biplot and the GE Biplot.

Outliers	Percentage of Outliers	The GGE Biplot			The GE Biplot		
		The magnitude of the outlier			The magnitude of the outlier		
		$k = 3$	$k = 10$	$k = 15$	$k = 3$	$k = 10$	$k = 15$
Scattered Outliers	2%	0.002	0.001	0.568	0.001	0.001	0.603
	5%	0.003	0.294	0.610	0.001	0.303	0.626
	10%	0.070	0.159	0.035	0.081	0.174	0.043
Single Environment (SE) Outliers	2%	0.001	0.012	0.179	0.002	0.018	0.194
	5%	0.001	0.067	0.214	0.001	0.066	0.250
	10%	0.001	0.031	0.049	0.001	0.048	0.054

Table 3a. The percentages of variance explained by the Biplot of the GE model.

Outlier	Percentage of Outliers	The magnitude of the outlier		
		$k = 3$	$k = 10$	$k = 15$
Without Outlier	0%	66.15		
Scattered Outliers	2%	63.89	60.38	47.80
	5%	58.25	48.34	40.93
	10%	51.01	42.39	48.03
Single Environment (SE) Outliers	2%	67.53	74.86	77.72
	5%	66.51	76.43	81.26
	10%	63.89	60.38	47.80

Table 3b. The percentages of variance explained by the Biplot of the GGE model.

Outlier	Percentage of Outliers	The magnitude of the outlier		
		$k = 3$	$k = 10$	$k = 15$
Without Outlier	0%	48.75		
Scattered Outliers	2%	55.32	64.29	48.52
	5%	48.07	47.6	40.96
	10%	39.65	41.75	46.22
Single Environment (SE) Outliers	2%	56.05	71.98	78.38
	5%	56.18	71.1	80.15
	10%	51.62	65.09	69.06

Now we turn to see whether outliers affect the percentage of total variance explained by the Biplot. Tables 3a and 3b show the percentage of total variance explained after the scattered and the single environment outliers by the Biplot of GE and GGE respectively. The GE Biplot seems to be affected by the scattered outlier since there was a noticeable drop in the percentage of variance explained for all number and value of the scattered outliers (table 3a). But for the GGE one, based on table 4b there is still higher percentage of variance explained at simulated data with small value and small number of scattered outliers.

On the other hand, in single environment outliers, again, there are strong indications that Biplot GE and GGE have a good robustness. Table 3 shows that in the presence of single environments it is followed by an increase in the percentage of total variance explained by the GGE Biplot, for both small and large outliers as well as for large and small percentage outliers. The exceptions only occur

for the GE Biplot with the data containing many outliers with any magnitude $k = 3, 10$, and also $k=15$. This shows us that Biplot GE fails to accommodate inflation variance by the presence of a single environmental outlier in large percentage.

4. Discussions: the GGE Biplot and its robustness

The RCIM Biplot of GGE model seems to hold a good robustness to the single environment outliers according to the MSE and also the Procrustes analysis. This robustness came out from the model of decomposing the interaction, i.e. the GGE model was accommodates the interaction terms in the G plus the pure interaction of G by E. When there is an increasing observation value by outliers in a single environment, the G effects from that “single environment” will increase as well as the GE effects, then the GGE model will immediately model the increase in the G + GE terms and decompose the GGE interactions by the Biplot, properly.

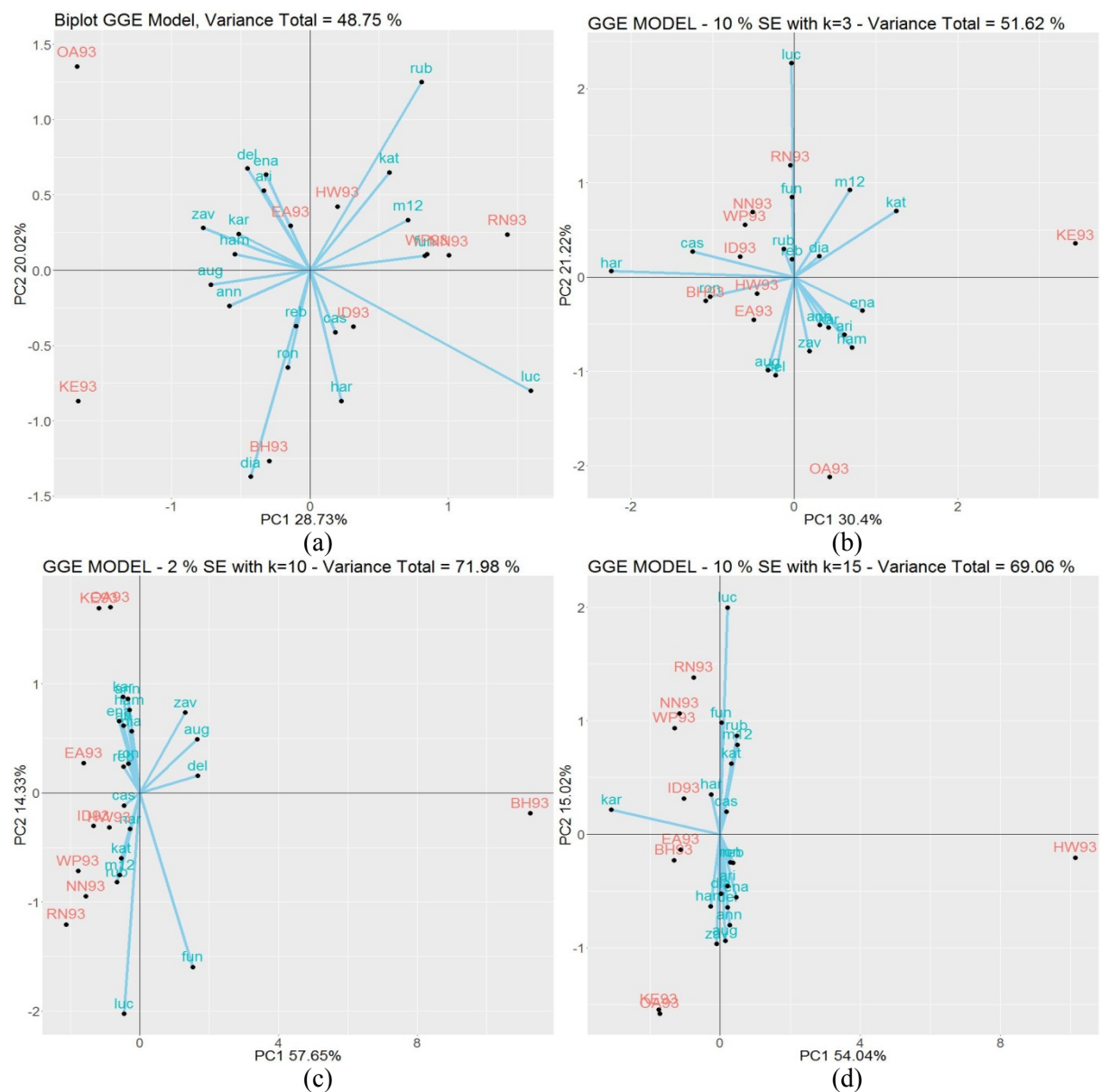


Figure 2. The GGE Biplot with (a) no outlier (original data), (b) with 10% SE outliers of $k = 3$, (c) 2% SE outliers of $k=10$, (d) 10% SE outliers of $k=15$.

As shown in Figure 2b, for the small value outliers but many, some environments increase in small magnitudes; therefore Biplot GGE produces a little change in configuration even it is very similar to the Biplot of original data (Figure 2a.). The total variance explained increase slightly in accordance with the addition of variance in some environments with small magnitudes. Conversely, if an increase only occurs in a particular environment with a large magnitude (2%, $k = 10$) then the increase only occurs in certain genotypes in particular environments resulting in a spike in the variance of the data. The GGE Biplot then modelled it accurately as Figure 2c. With very high value of % total variance explained, 71.98%, it shows the capability to accommodate the variance inflation.

Meanwhile, when the single environment outliers with large magnitude occur in many environments it is not necessarily resulting in a variance inflation, as followed by a rising shift mean in several environments simultaneously, so that the variance of the data with these outliers does not uphill dramatically. Thus, the Biplot will model a moderate variance rather than a variance with large inflation.

This is evident in low percentage of total variance explained by the GGE Biplot and even more by the GE Biplot. However, this decrease in total variance explained by the GGE was not lower than the original data, not as worse as the GE Biplot was.

5. Concluding Remark

The GGE Biplot in RCIM model has good robustness to the single environment outliers according to the MSE and also the Procrustes analysis. This robustness came out from the model of decomposing the interaction, i.e. the GGE Biplot was accommodates the interaction terms in the G plus the pure interaction of G by E. The GE Biplot fails to accommodate inflation variance by the presence of a single environmental outlier with large percentage. When the single environment outliers with large magnitude occur in many environments, it is not necessarily resulting in a variance inflation. In this situation, the Biplot will be expected to model a moderate variance rather than a variance with large inflation, but the GE Biplot faces difficulties to hold the total percentage of the variance explained.

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